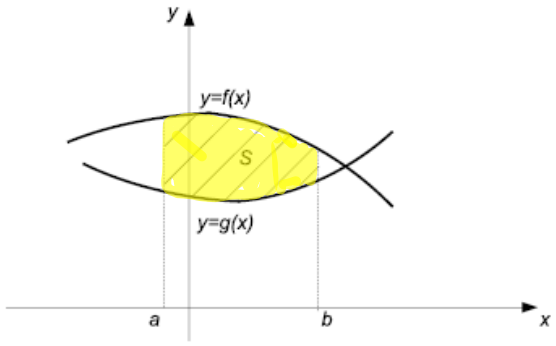
Section 6.6. **Area between two curves.**

Consider the region S that lies between two curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$.



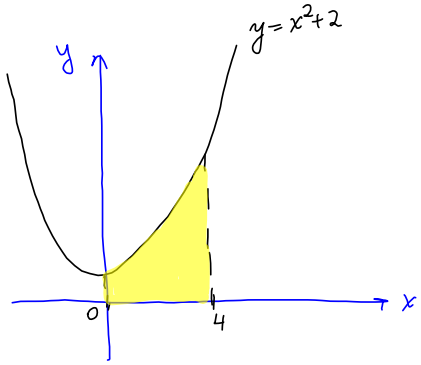
The area of the region S is

$$A = \int_a^b [f(x) - g(x)]dx$$

Example 1. Find the area of the region bounded by

1. $y = x^2 + 2$, $y = 0$, $x = 0$, $x = 4$

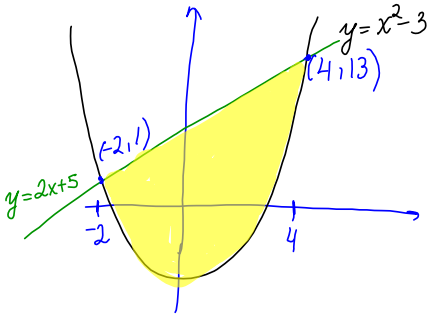
$y = 0$: x-axis



$$A = \int_0^4 (x^2+2-0)\,dx$$

$$= \int_0^4 (x^2+2)\,dx = \frac{88}{3}$$

2. $y = x^2 - 3$, $y = 2x + 5$



Points of intersection.

$x^2 - 3 = 2x + 5$

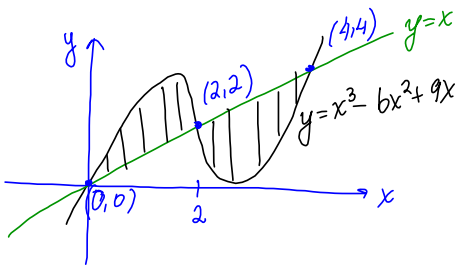
$x_1 = -2$, $x_2 = 4$

$-2 \leq x \leq 4$

$$A = \int_{-2}^{4} [\,2x+5-(x^2-3)\,]\,dx$$

$$= \int_{-2}^{4} (2x - x^2 + 8)\,dx = 36$$

3. $y = x^3 - 6x^2 + 9x,\ y = x.$

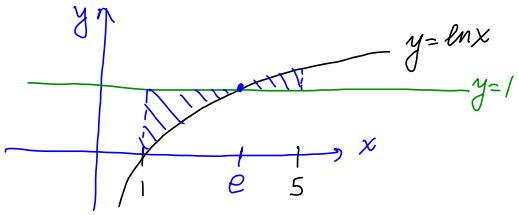


$$A = \int_0^2 \left[x^3 - 6x^2 + 9x - x \right] dx + \int_2^4 \left[x - (x^3 - 6x^2 + 9x) \right] dx$$

$$= \int_0^2 (x^3 - 6x^2 + 8x)\,dx + \int_2^4 (-x^3 + 6x^2 - 8x)\,dx$$

$$= \boxed{8}$$

4. $y = \ln x,\ y = 1,\ x = 1,\ x = 5.$

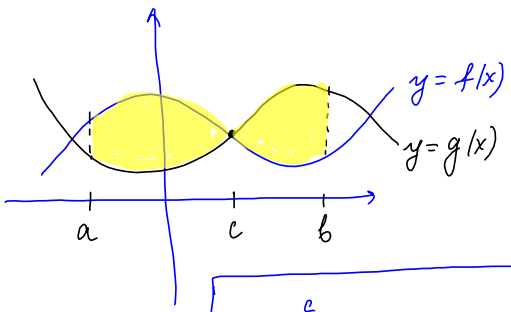


Points of intersection:

$\ln x = 1$, $\boxed{x = e}$

$$A = \int_1^e (1 - \ln x)\,dx + \int_e^5 (\ln x - 1)\,dx$$

$$= \boxed{1.48}$$



Find the area of the region between $y = f(x)$ and $y = g(x)$ for $a \le x \le b$

$$\boxed{A = \int_a^c \left(f(x) - g(x) \right) dx + \int_c^b \left(g(x) - f(x) \right) dx}$$