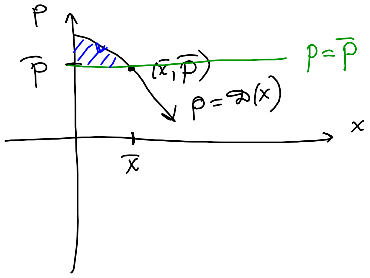
Section 6.7 **Additional Applications of the Integral.**

Consumers' and producers' surplus.

Let $p = D(x)$ be the price-demand equation for a product, where x is the number of units of the product that consumers will purchase at a price of \$p per unit. Suppose that $\bar{p}$, is the current price and $\bar{x}$ is the number of units that can be sold at that price.



Definition. Consumers' Surplus If $(\bar{x}, \bar{p})$ is a point on the graph of the price-demand equation $p = D(x)$ for a particular product, then the consumers' surplus CS at a price level of $\bar{p}$ is

$$\boxed{CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx} = \text{area of the shaded region}$$

which is the area between $p = \bar{p}$ and $p = D(x)$ from $x = 0$ to $x = \bar{x}$.

The consumers' surplus represents the total savings to consumers who are willing to pay more that $\bar{p}$ for the product, but are still able to buy the product for $\bar{p}$.

Example 1. Find the consumers' surplus at a price level of $\boxed{\bar{p} = \$120}$ for the price-demand equation

$$p = D(x) = 200 - 0.02x$$

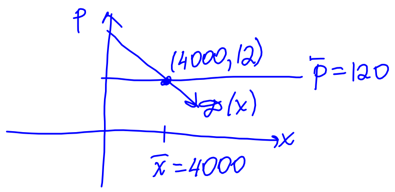
$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]\,dx$$

Point of intersection

$200 - 0.02x = 120$

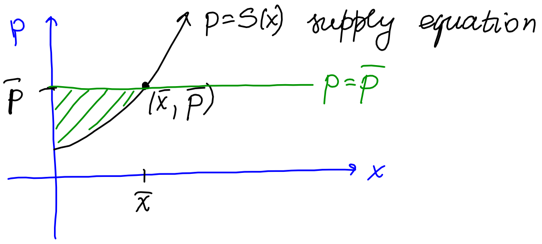
$x = 4000$

$\boxed{\bar{x} = 4000}$



$$CS = \int_0^{4000} [200 - 0.02x - 120]\,dx = \boxed{160{,}000}$$

Definition. Producers' surplus.



If $(\bar{x}, \bar{p})$ is a point on the graph of the price-supply equation $p = S(x)$, then the producers' surplus PS at a price level of $\bar{p}$ is

$$\boxed{PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx} = \text{area of the shaded region}$$

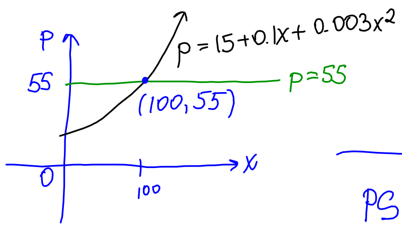
which is the area between $p = \bar{p}$ and $p = S(x)$ from $x = 0$ to $x = \bar{x}$.

The producers' surplus represents the total gain to producers who are willing to supply units at a lower price that $\bar{p}$ but are still able to supply units at $\bar{p}$.

Example 2. Find the producers' surplus at a price level of $\bar{p} = \$55$ for the price-supply equation

$$p = S(x) = 15 + 0.1x + 0.003x^2$$

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]\,dx$$



Point of intersection:

$$15 + 0.1x + 0.003x^2 = 55$$

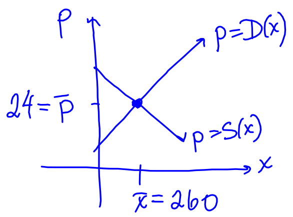
$$x = \boxed{100 = \bar{x}}$$

$$PS = \int_0^{100} [55 - 15 - 0.1x - 0.003x^2]\,dx = \boxed{2500}$$

If $p = D(x)$ and $p = S(x)$ are the price-demand and price-supply equations, respectively, for a product and if $(\bar{x}, \bar{p})$ is the point of intersection of these equations, then $\bar{p}$ is called the **equilibrium price** and $\bar{x}$ is called the **equilibrium quantity**. If the price stabilizes at the equilibrium price $\bar{p}$, then this is the price level that will determine both the consumers' surplus and the producers' surplus.

Example 3. Find the equilibrium price and then find the consumers' surplus and producers' surplus at the equilibrium price level if

$$p = D(x) = 50 - 0.1x, \quad p = S(x) = 11 + 0.05x.$$



$\bar{x}$ is the equilibrium quantity

$\bar{y}$ is the equilibrium price.

$\boxed{\bar{x} = 260, \quad \bar{p} = 24}$

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]\,dx = \int_0^{260} (50 - 0.1x - 24)\,dx = \boxed{3380 = CS}$$

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]\,dx = \int_0^{260} (24 - 11 - 0.05x)\,dx = \boxed{1690 = PS}$$