MATH 142–511, 516, 517 Spring 2010

Sample problems for the Final

1. How many CDs would a record company have to make and sell to break even if the fixed costs are \$18000, variable costs are \$5.20 per CD, and the CDs are sold to retailers for \$7.60 each?

2. Trunsville Utilities uses the following rates to compute the monthly cost of natural gas for residential customers. Write a piecewise definition for the cost of consuming x CCF (cubic hundred feet) of natural gas.

Charges per month
\$0.7675 per CCF for the first 50 CCF
\$0.6400 per CCF for the next 150 CCF
\$0.6130 per CCF for all over 200 CCF

3. Find the domain of the function

$$f(x) = \begin{cases} \sqrt{x-5}, & \text{if } 5 \le x \le 10 \\ \frac{1}{2x+5}, & \text{if } 10 < x \le 20 \\ 5x, & \text{if } x > 20 \end{cases}$$

4. The revenue and cost functions for a particular product are given below. The cost and revenue are given in millions of dollars, and x represents the number of units (in thousands).

$$R(x) = -64x^2 + 4400x$$
$$C(x) = 250x + 5200$$

At what production level(s), rounded to the nearest whole unit, will the company break even on this product?

5. Solve the following equations for x:

   (a) $2^{6x} = 8^{x^2+1}$

   (b) $(5-x)^5 = (2x-1)^5$

   (c) $5^x = 14$

   (d) $\log_2(x+2) + \log_2(2) = \log_2 21$

   (e) $\log_4 x = 3$

6. Suppose that \$2500 is invested at 7% compounded quarterly. How much money will be in the account in 5 years?

7. In its first 10 years the Gabelli Growth Fund produced an average annual return of 21.36%. Assume that money invested in this fund continues to earn 21.36% compounded annually. How long will it take money invested in this fund to double?

8. Evaluate each limit:

   (a) $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$

   (b) $\lim_{x \to \infty} \dfrac{2x^3 + x^2 + 5}{x - x^3}$

   (c) $\lim_{x \to 1^-} \dfrac{|x - 1|}{x - 1}$

9. Determine where the function

$$f(x) = \begin{cases} 1 + x, & \text{if } x \leq 2 \\ 6 - x, & \text{if } x > 2 \end{cases}$$

   is continuous.

10. Find all vertical and horizontal asymptotes for the function

$$f(x) = \frac{(2x-3)(x+2)(x^2-1)}{(x-4)(x+3)x(x-1)}$$

11. Let

$$p = 25 - 0.01x \qquad \text{and} \qquad C(x) = 2x + 9000$$

   where $0 \leq x \leq 2500$, be the price-demand equation and cost function, respectively, for the manufacture of umbrellas.

   (a) Find the exact cost of producing the 31st umbrella. Use the marginal cost to approximate the cost of producing the 31st umbrella.

   (b) Find the marginal revenue and the marginal average revenue functions.

   (c) Find the average profit per umbrella if 50 umbrellas is produced. Find the marginal average profit at a production level of 50 umbrellas. Estimate the average profit per umbrella if 51 umbrella is produced.

12. A bank offers a 10-year certificate of deposit (CD) that earns 4.15% compounded continuously.

   (a) If \$10000 is invested in this CD, how much will it be worth in 10 years?

   (b) How long will it take for the account to be worth \$18000?

13. A note will pay \$25000 at maturity 10 years from now. How much should you willing to pay for the note now if money is worth 5% compounded continuously?

14. At what nominal rate compounded continuously must money be invested to double in 8 years?

15. Find the equation of the tangent line to the graph of the function $f(x) = \ln(1 - x^2 + 2x^4)$ at the point where $x = 1$.

16. Find the value(s) of x where the tangent line to the graph of the function $y = 5e^{x^2-4x+1}$ is horizontal.

17. Find each derivative

(a) $\frac{d}{dx}\log_3(\sqrt[4]{4x^3+5x+7})$

(b) $\frac{d}{dx}8^{1-2x^3}$

(c) $\frac{d}{dx}\frac{3x^2}{(x^2+5)^3}$

(d) $\frac{d}{dx}[(x^2+x-3)e^{2x+3}]$

18. Given the price-demand equation

$$0.02x + p = 60$$

(a) Find the elasticity of demand $E(p)$.

(b) For which values of p is demand elastic?

(c) If $p = \$10$ and the price is increased by 5%, what is the approximate change in demand?

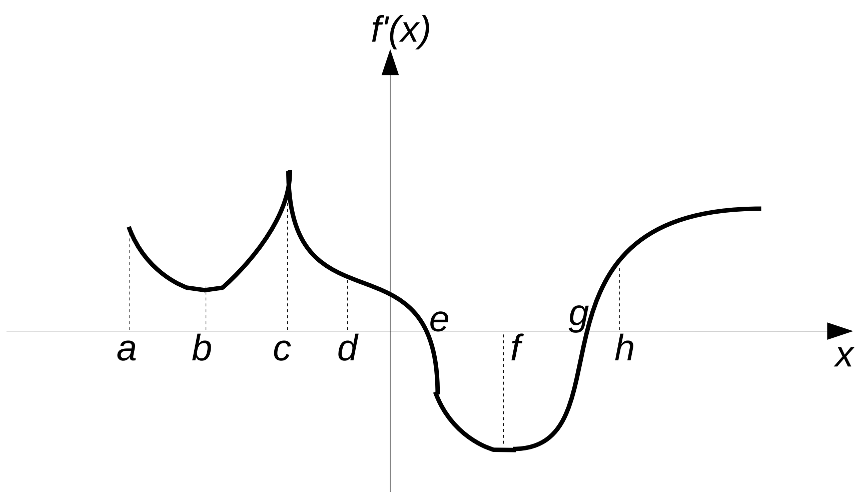
(d) If $p = \$40$ and the price is decreased, will revenue increase or decrease?

19. Find $f''(x)$ for the functions

(a) $f(x) = x^2(2x^3-5)^4$

(b) $f(x) = \frac{2}{x} - \frac{6}{x^3}$

20. Given the graph of the derivative $f'(x)$ of the function $y = f(x)$.



(a) Find the intervals on which f is increasing, decreasing.

(b) Find x-coordinates of the critical points for the function f.
(c) Find the intervals on which f is concave upward, concave downward.
(d) Find x-coordinates of the inflection points for f.

21. Given the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 3x + 4$.

    (a) Find critical values of $f(x)$.
    (b) Find intervals on which $f(x)$ is increasing and decreasing.
    (c) Find local extrema for $f(x)$.
    (d) Find intervals on which $f(x)$ is concave upward and concave downward.
    (e) Find all inflection points of $f(x)$.

22. Find the absolute maximum and absolute minimum for the function $f(x) = \sqrt{9 - x^2}$ on the interval $[-1, 2]$.

23. Find the absolute maximum and minimum for the function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

24. What are the dimensions of the rectangular field of 20000 square feet that will minimize the cost of fencing if one side costs three times as much per unit length as the other three?

25. A 300-room hotel in Las Vegas is filled to capacity every night at $100 a room. For each $2 increase in rent, 5 fewer rooms are rented. If each rented room costs $10 service per day, how much should be management charge for each room to maximize gross profit? What is the maximum gross profit?

26. Find the following indefinite integrals:

    (a) $\displaystyle\int \frac{1}{\sqrt{x}} e^{\sqrt{x}}\, dx$
    (b) $\displaystyle\int \frac{x^3 - 3}{x^4 - 12x + 3}\, dx$
    (c) $\displaystyle\int \frac{(\ln x)^2}{x}\, dx$
    (d) $\displaystyle\int \frac{x}{(5 - 2x^2)^5}\, dx$
    (e) $\displaystyle\int \frac{x}{\sqrt{3 - x}}\, dx$
    (f) $\int e^{2x-1}\, dx$

27. The weekly marginal revenue from the sale of x pairs of tennis shoes is given by

$$R'(x) = 40 - 0.002x + \frac{200}{x+1}, \qquad R(0) = 0,$$

where $R(x)$ is revenue in dollars. Find the revenue function $R(x)$. Find the revenue from the sale of 1000 pairs of shoes.

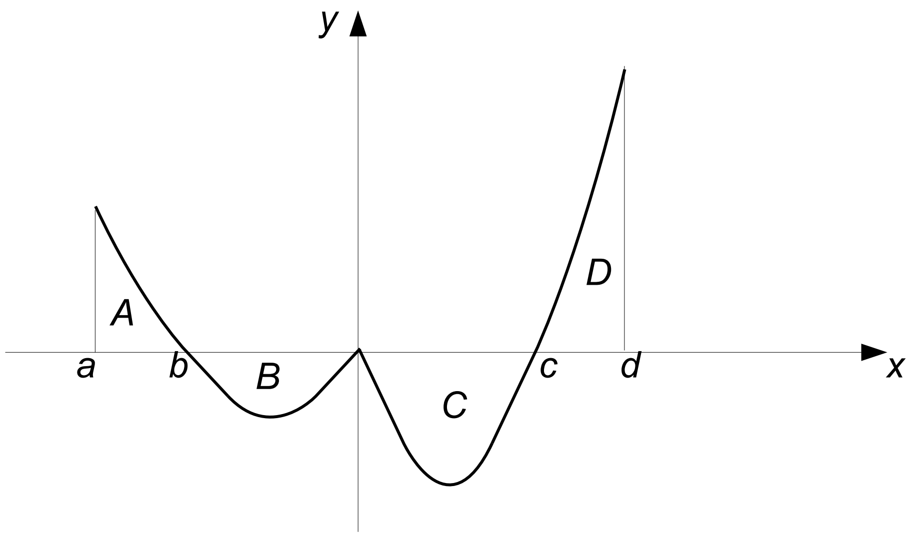
28. Calculate the following Riemann sum for the function $f(x) = x^2 - 9x + 7$. Partition the interval $[-1, 3]$ into four subintervals of equal length. For each subinterval $[x_k, x_{k+1}]$ let c_k be the midpoint.

29. Evaluate each of the definite integrals:

   (a) $\int_0^A 32(x^2+1)^7 x\,dx \quad (A > 0)$

   (b) $\int_B^2 \left(5x - 4\frac{x^2}{\sqrt[4]{x^3}}\right) dx, \quad (0 < B < 2)$

30. Calculate the definite integral $\int_b^d f(x)dx$ by referring to the figure and list of area values below.



if of $A = 1.4$, area of $B = 2.4$, area of $C = 3.1$, area of $D = 2.1$.

31. Find the average value of the function $f(x) = 4x - 3x^2$ over the interval $[-2, 2]$.

32. Find the area of the region bounded by:

   (a) $y = 3 - x^2$, $y = 2x^2 - 4x$
   (b) $y = x^3$, $y = 4x$
   (c) $y = -x^2 - 2x$, $y = 0$, $x = -2$, $x = 1$.

33. Find the consumers' surplus and producers' surplus at the equilibrium price level for the given price-demand and price-supply equations.

$$p = D(x) = 70 - 0.2x$$
$$p = S(x) = 13 + 0.0012x^2$$

Round all values to the nearest integer.

34. Let $f(x, y) = 2x - 3y + 14$ and $g(x, y) = \dfrac{10}{x^2 + 4y}$. Find $f(2, -3) - 4g(-1, 2)$.

35. Weston Publishing publishes a deluxe edition and a standard edition of its English language dictionary. Weston's management estimates that the number of deluxe editions demanded is x copies/day and the number of standard editions demanded is y copies/day when the unit prices are

$$p = 20 - 0.005x - 0.001y$$
$$q = 15 - 0.001x - 0.003y$$

dollars, respectively. Find the daily total revenue function $R(x, y)$. Evaluate $R(7, 3)$.

36. Find the cross-section of the surface $z = 10x + 4xy + 15y^2 - 6x^2 + 5$ produced by the cutting it with the planes $x = 4$, $y = 2$.

37. Find Find f_x, f_y, f_{xx}, f_{xy}, and f_{yy} for the functions:

   (a) $f(x, y) = \sqrt{2x - y^2}$
   (b) $f(x, y) = e^{x\sqrt{y}}$
   (c) $f(x, y) = \ln(x^3 - y^2)$

38. Find the local extrema for the function $f(x, y) = -x^2 - y^2 + 2x + 4y + 5$.

39. Average global temperatures from 1885 to 2005 are given in the table

Year	**^{o}F**	**Year**	**^{o}F**
1885	56.65	1955	57.06
1895	56.64	1965	57.05
1905	56.52	1975	57.04
1915	56.57	1985	57.36
1925	56.74	1995	57.64
1935	57.00	2005	58.59
1945	57.13		

   (a) Find the least squares line for the data, using $x = 0$ for 1885.
   (b) Use the least squares line to estimate the average global temperature in 2085.