Math 142 – 511, 516, 517, Spring 2010

Lecture 4.

1/28/2010

Homeworks #1, #2, #3, and #4 are **due Thursday, Jan. 28, 11:55 PM.**

Homework #5 (Section 2.5)
Homework #6 (Section 3.1)
Homework #7 (Section 3.2)
are **due Thursday, Feb. 4, 11:55 PM.**

Help Sessions:

Mondays	7:00PM - 9:00PM	BLOC 117
Tuesdays	7:00PM – 9:00PM	BLOC 164
Thursdays	7:00PM – 9:00PM	BLOC 164

Section 2.5 **Logarithmic functions**

Inverse functions.
Definition. A function f is said to be **one-to-one** if each range value corresponds to exactly one domain value.
Horizontal line test. A function is one-to-one if and only if no horizontal line intersects its graph more that once.
Definition. If f is a one-to-one function, then the **inverse** of f is the function formed by interchanging the independent and dependent variables for f.
Note: if f is not one-to-one, then f **does not have an inverse.**

Logarithmic functions.
The inverse of an exponential function is called a **logarithmic function.** For $b > 0$ and $b \neq 1$,

$$y = \log_b x \text{ is equivalent to } x = b^y.$$

The **log to the base b of x** is the exponent to which b must be raised to obtain x. The **domain** of the logarithmic function is $(0, \infty)$ and the **range** of the logarithmic function is $(-\infty, \infty)$. If $b > 1$, then $y = \log_b x$ increases as x increases; if $0 < b < 1$, then $y = \log_b x$ is decreases as x increases.

Example 1. Rewrite

$$\log_2 16 = 4$$

in an equivalent exponential form.

Example 2. Change

$$125 = 5^3$$

to an equivalent logarithmic form.

$$\log_e x = \ln x \quad \log_{10} x = \log x$$

If $x, y > 0$ and b, c, and k are constants, $b > 0$, $b \neq 1$, $c > 0$, $c \neq 1$, then

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b(b^x) = x$
4. $b^{\log_b x} = x$
5. $\log_b(xy) = \log_b x + \log_b y$
6. $\log_b \frac{x}{y} = \log_b x - \log_b y$
7. $\log_b(x^k) = k\log_b x$
8. $\log_b x = \dfrac{\log_c x}{\log_c b}$
9. $\log_b x = \log_b y \quad \text{if and only if} \quad x = y$

Example 3. Solve the following equations:

a) $\log_{25} x = \dfrac{1}{2}$

b) $\log_5 x = \dfrac{2}{3}\log_5 8 + \dfrac{1}{2}\log_5 9 - \log_5 6$

c) $\log_2(x-1) = 2 + \log_2(x+4)$

d) $1.03^x = 2.43$

Chapter 3. **Limits and Derivative**
Section 3.1 **Introduction to limits**

Definition We write

$$\lim_{x \to c} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to c$$

and say "the limit of $f(x)$, as x approaches c, if the functional value $f(x)$ is close to the single number L whenever x is close, but not equal, to c (on both sides of c).

Definition We write

$$\lim_{x \to c^-} f(x) = K$$

and call K the **limit from the left** or the **left-hand limit** if $f(x)$ is close to K whenever x is close to, but to the left of, c on the real number line.

We write

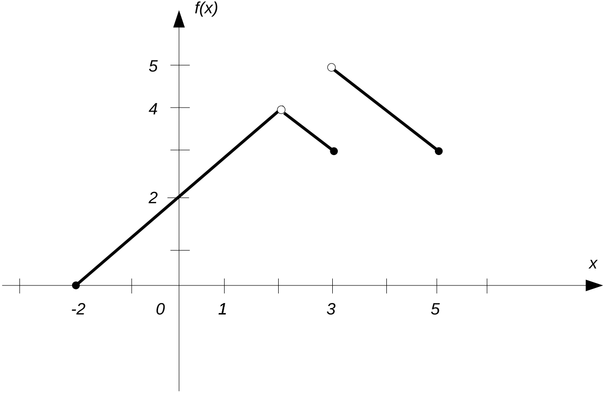
$$\lim_{x \to c^+} f(x) = L$$

and call L the **limit from the right** or the **right-hand limit** if $f(x)$ is close to L whenever x is close to, but to the right of, c on the real number line.

If no direction is specified in a limit statement, we will always assume that the limit is **two-sided** or **unrestricted**.

$$\lim_{x \to c} f(x) = L \text{ if and only if } \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

Example 4. Given the graph of the function f



Find: a) $\lim_{x \to 1} f(x)$ b) $\lim_{x \to 2^+} f(x)$ c) $\lim_{x \to 2^-} f(x)$

d) $\lim_{x \to 3^+} f(x)$ e) $\lim_{x \to 3^-} f(x)$