Math 142 – 511, 516, 517, Spring 2010

Lecture 5.

2/2/2010

Homework #5 (Section 2-5)
Homework #6 (Section 3-1)
Homework #7 (Section 3-2)
are **due Thursday, Feb. 4, 11:55 PM.**

Quiz 3 will be held oh Thursday, Feb. 4. It will cover sections 2-5, 3-1, 3-2.

Chapter 3. **Limits and the Derivative**
Section 3-1 **Introduction to limits**

Definition We write

$$\lim_{x \to c} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to c$$

and say "the limit of $f(x)$, as x approaches c, if the functional value $f(x)$ is close to the single number L whenever x is close, but not equal, to c (on both sides of c).

Definition We write

$$\lim_{x \to c^-} f(x) = K$$

and call K the **limit from the left** or the **left-hand limit** if $f(x)$ is close to K whenever x is close to, but to the left of, c on the real number line.

We write

$$\lim_{x \to c^+} f(x) = L$$

and call L the **limit from the right** or the **right-hand limit** if $f(x)$ is close to L whenever x is close to, but to the right of, c on the real number line.

If no direction is specified in a limit statement, we will always assume that the limit is **two-sided** or **unrestricted**.

Theorem.

$$\lim_{x \to c} f(x) = L \text{ if and only if } \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

Limit laws Suppose that k is a constant and the limits $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist. Then

1. $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
2. $\lim_{x \to c} [f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$
3. $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$
4. $\lim_{x \to c} f(x)g(x) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$
5. $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ if $\lim_{x \to c} g(x) \neq 0$
6. $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x) \right]^n$ where n is a positive integer
7. $\lim_{x \to c} k = k$ 8. $\lim_{x \to c} x = c$
9. $\lim_{x \to c} x^n = c^n$ where n is a positive integer
10. $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$ where n is a positive integer
11. $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$ where n is a positive integer

Example 1. Find each limit if it exists.

(a) $\lim_{x \to 5} (3x + 6)$ (b) $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x + 1}$

(c) $\lim_{x \to 2} x(x + 1)$ (d) $\lim_{x \to -1} \sqrt{4 - 5x}$

Limits of polynomial and rational functions.
(a) $\lim_{x \to c} f(x) = f(c)$ if f is a polynomial function
(b) $\lim_{x \to c} r(x) = r(c)$ if r is a rational function with a nonzero denominator at $x = c$.

Example 2. Find each limit if it exists.
(a) $\lim_{x \to 1} (x^5 - 3x^3 + 5x + 4)$

(b) $\lim_{x \to 3} \dfrac{x^2 + 2}{x^3 + 4}$

(c) $\lim_{x \to 2} \dfrac{x|x - 2|}{x - 2}$

Indeterminate form. If $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$, then $\lim_{x \to c} \frac{f(x)}{g(x)}$ is said to be **indeterminate** , or, more specifically, a **0/0 indeterminate form.**

Limit of a quotient. If $\lim_{x \to c} f(x) = L$, $L \neq 0$, and $\lim_{x \to c} g(x) = 0$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} \text{ does not exist .}$$

Example 3. Let $f(x) = \dfrac{x^2 + x - 6}{x^2 + 2x - 3}$. Find

(a) $\lim_{x \to -3} f(x)$,

(b) $\lim_{x \to 1} f(x)$

Section 3-2 **Continuity.**

Definition. A function f is **continuous at the point** $x = c$ if

1. $\lim_{x \to c} f(x)$ exists 2. $f(c)$ exists 3. $\lim_{x \to c} f(x) = f(c)$.

A function is **continuous on the open interval** (a, b) if f is continuous at each point on the interval.

If one or more of the three conditions in the definition fails, the function is **discontinuous** at $x = c$.

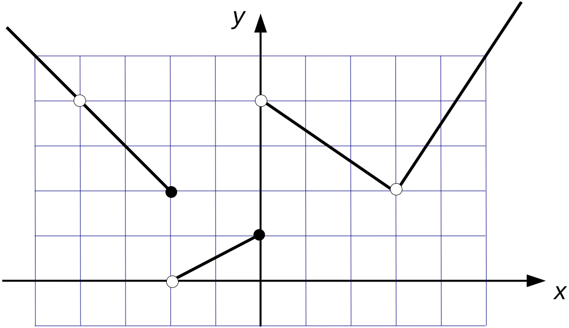
Definition A function f is **continuous from the right at a number** c if

$$\lim_{x \to c^+} f(x) = f(c),$$

f is **continuous from the left at a number** c if

$$\lim_{x \to c^-} f(x) = f(c).$$

Example 4. Discuss the continuity of the function whose graph is



Continuity properties.

If two functions are continuous on the same interval, then their sum, difference, product, and quotient are continuous on the same interval, except for values of x that make a denominator 0.

Continuity properties of some specific functions.

1. A constant function $f(x) = k$, where k is a constant, is continuous on $(-\infty, \infty)$.
2. For n a positive integer, $f(x) = x^n$ is continuous on $(-\infty, \infty)$.
3. A polynomial function is continuous on $(-\infty, \infty)$.
4. A rational function is continuous for all x except those values that make a denominator 0.
5. For n an odd positive integer, $\sqrt[n]{f(x)}$ is continuous wherever $f(x)$ is continuous.
6. For n an even positive integer, $\sqrt[n]{f(x)}$ is continuous wherever $f(x)$ is continuous and nonnegative.

Example 5. Determine where each function is continuous.

(a) $f(x) = 3x + 2$ (b) $f(x) = x^{25} - x^3 + 4$

(c) $f(x) = \dfrac{x^2 + 4}{4 - 25x^2}$ (d) $f(x) = \begin{cases} x + 4, & \text{if } x \leq 3 \\ 2x - 1, & \text{if } x > 3 \end{cases}$