Math 142 – 511, 516, 517, Spring 2010

Lecture 6.

2/4/2010

Homework #5 (Section 2-5)
Homework #6 (Section 3-1)
Homework #7 (Section 3-2)
are **due Thursday, Feb. 4, 11:55 PM.**

Week-in-Review on Sunday, Feb. 7, will be held at 11am – 1pm in BLOC 102.

Homework #8 (Section 3-3)
Homework #9 (Section 3-4)
are **due Thursday, Feb. 11, 11:55 PM.**

Chapter 3. **Limits and the Derivative**
Section 3-2 **Continuity.**

Definition. A function f is **continuous at the point** $x = c$ if

1. $\lim_{x \to c} f(x)$ exists 2. $f(c)$ exists 3. $\lim_{x \to c} f(x) = f(c)$.

A function is **continuous on the open interval** (a, b) if f is continuous at each point on the interval.

If one or more of the three conditions in the definition fails, the function is **discontinuous** at $x = c$.

If two functions are continuous on the same interval, then their sum, difference, product, and quotient are continuous on the same interval, except for values of x that make a denominator 0.

Solving inequalities by using continuity properties.

Sign properties on an interval (a, b)**.** If f is continuous on (a, b) and $f(x) \neq 0$ for all $x \neq 0$ for all x in (a, b), then either $f(x) > 0$ for all x in (a, b) or $f(x) < 0$ for all x in (a, b).

Constructing sign charts. Given a function f.

Step 1. Find all partition numbers. That is,
(A) Find all numbers such that f is discontinuous.
(B) Find all numbers such that $f(x) = 0$.

Step 2. Plot the numbers found in step on a real-number line, dividing the number line into intervals.

Step 3. Select a test number in each open interval determined in step 2, and evaluate $f(x)$ at each test number to determine whether $f(x)$ is positive $(+)$ and negative (–) in each interval.

Step 4. Constructing a sign chart, using the real-number line in step 2. This will show the sign of $f(x)$ on each open interval.

Example 1. Solve the inequality

$$\frac{x^2 + 5x}{x - 3} < 0.$$

Section 3-3 **Infinite limits and limits at infinity.**

If a function f either increases or decreases without bound as x approaches a, we say that the

$$\lim_{x \to a} f(x) = \infty.$$

Example 2. Find each of the following limits

(a) $\displaystyle\lim_{x \to 2} \frac{1}{x-2}$,

(b) $\displaystyle\lim_{x \to 2} \frac{1}{(x-2)^2}$.

Definition. The vertical line $x = a$ is a **vertical asymptote** for the graph of $y = f(x)$ if

$$f(x) \to \infty \text{ or } f(x) \to -\infty \text{ as } x \to a^+ \text{ or } x \to a^-.$$

If $f(x) = n(x)/d(x)$ is a rational function, $d(c) = 0$ and $n(c) \neq 0$, then the line $x = c$ is a vertical asymptote of the graph of f.

Example 3. Find vertical asymptotes of the function
$$\frac{3x+2}{x^3+x^2-2x}.$$

The symbol ∞ also can be used to indicate that an independent variable is increasing or decreasing without bound. We write $x \to \infty$ to indicate that x increases without bound through positive values and $x \to -\infty$ to indicate that x decreases without bound through negative values.

Definition. A line $y = b$ is a horizontal asymptote of the graph of $y = f(x)$ if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

Limits of power functions at infinity. If p is a positive real number and k is any real constant, then

1. $\lim_{x \to -\infty} \frac{k}{x^p} = 0$ 2. $\lim_{x \to \infty} \frac{k}{x^p} = 0$

3. $\lim_{x \to -\infty} kx^p = \pm\infty$ 4. $\lim_{x \to \infty} kx^p = \pm\infty$

provided that x^p is a real number for negative values for x. The limits in 3 and 4 will be either $-\infty$ or ∞, depending on k and p.

Limits of polynomial functions at infinity. If

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0, \quad a_n \neq 0, \quad n \geq 1$$

then

$$\lim_{x \to \infty} p(x) = \lim_{x \to \infty} a_n x^n = \pm\infty$$

and

$$\lim_{x \to -\infty} p(x) = \lim_{x \to -\infty} a_n x^n = \pm\infty.$$

Each limit will be either ∞ or $-\infty$, depending on a_n and n.
Polynomials of degree 1 or greater never have horizontal asymptotes.

Limits of rational functions at infinity.

$$\lim_{x\to\infty} \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0} = \lim_{x\to\infty} \frac{a_n x^n}{b_m x^m} =$$

$$\begin{cases} \frac{a_n}{b_m}, & \text{if } n = m \\ 0, & \text{if } n < m \\ \infty, & \text{if } n > m \end{cases}$$

A rational function can have at most one horizontal asymptote.

Example 4. Evaluate the limit.

(a) $\lim_{x\to\infty} \frac{7x^3 + 4x}{2x^3 - x^2 + 3}$ (b) $\lim_{x\to\infty} \frac{x+4}{x^3 - 3}$

(c) $\lim_{x\to\infty} \frac{x^2 - 3x + 1}{2x + 3}$

Example 5. Find all the asymptotes of the function

$$f(x) = \frac{2x^2 + 3x - 2}{x^2 - x - 2}.$$