Math 142, 511, 516, 517, Spring 2010

Lecture 7.

2/9/2010

Homework #8 (Section 3-3)
Homework #9 (Section 3-4)
are **due Thursday, Feb. 11, 11:55 PM.**

Test 1 will be held on Thursday, Feb. 11. It will cover sections 2.2, 2.3 (what we did in class), 2.4, 2.5, 3.1–3.5.

How to clear your calculator: [2ND] [+], pick the option [7:Reset], cursor right to [ALL], [2:RESET]

Section 3-4 **The Derivative**

Example 1. The profit (in dollars) from the sale of x car seats for infants is given by

$$P(x) = 45x - 0.025x^2 - 5000, \qquad 0 \leq x \leq 2400$$

Find the average change in profit if production is changed from 800 sets to 850 seats.

Definition. For $y = f(x)$, the **average rate of change from** $x = a$ **to** $x = a + h$ is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \qquad h \neq 0.$$

The expression

$$\frac{f(a+h) - f(a)}{h}$$

is called the **difference quotient**.

Example 2. Suppose an object moves along the y-axis so its location is $y = x^2 + x$ at time x (y is in meters and x is in seconds). Find:

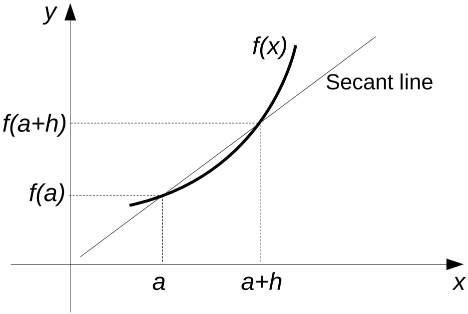
1. The average velocity (the average rate of change of y with respect to x) for x changing from 1 to 3 seconds.
2. The average velocity for x changing from 1 to $1 + h$ seconds.
3. The instantaneous velocity at $x = 1$ seconds.

Definition. For $y = f(x)$, the **instantaneous rate of change at** $x = a$ is

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

if the limit exists.

Slope of a secant line.



Definition. Given $y = f(x)$, the **slope of the graph** at the point $(a, f(a))$ is given by

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided that the limit exists. The slope of the graph is also the **slope of the tangent line** at the point $(a, f(a))$.

Definition For $y = f(x)$ we define the **derivative of f at x**, denoted by $f'(x)$, to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{if the limit exists}$$

If $f'(x)$ exists for each x in the open interval (a, b), then f is said to be **differentiable** over (a, b).
(Differentiability from the left or from the right is defined by using $h \to 0^-$ or $h \to 0^+$, respectively, in place $h \to 0$ in the preceding definition).
The process of finding the derivative of a function is called **differentiation**.
The derivative of a function f is a new function f'.
Interpretations of the derivative.

1. Slope of the tangent line.
2. Instantaneous rate of change.
3. Velocity.

How to find the derivative of a function f:

1. Find $f(x+h)$
2. Find $f(x+h) - f(x)$
3. Find $\dfrac{f(x+h) - f(x)}{h}$.
4. Find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Example 3. Find $f'(x)$ for $f(x) = x + 4$.

Example 4. Where is the function $f(x) = |x - 2|$ differentiable?

When is the function not differentiable at $x = a$?

1. f has a "corner" at a
2. f is discontinuous at a
3. the curve $y = f(x)$ has a vertical tangent line at $x = a$

Section 3-5 **Basic differentiation properties**

Table of derivatives

1.$(C)' = 0$, C is a constant,

2.$(x)' = 1$,

3.$(x^2)' = 2x$,

4.$(x^n)' = nx^{n-1}$,

Differentiation formulas

Suppose c is a constant and both functions $f(x)$ and $g(x)$ are differentiable, then

(a) $(cf(x))' = cf'(x)$,

(b) $(f(x) + g(x))' = f'(x) + g'(x)$,

(c) $(f(x) - g(x))' = f'(x) - g'(x)$.

Example 5. Differentiate each function.
(a) $f(x) = x^5 - 4x^3 + 2x - 3$
(b) $f(x) = 3x^{2/3} - 2x^{5/2} + x^{-3}$
(c) $f(x) = x^2\sqrt[3]{x^2}$

Example 6. Find the equation to the tangent line to the curve $y = x + \sqrt{x}$ at the point (1,2)