Math 142, 511, 516, 517, Spring 2010

Lecture 8.

2/16/2010

Homework #10 (Section 3-5)
Homework #11 (Section 3-7)
are **due Thursday, Feb. 18, 11:55 PM.**

Table of derivatives

1.$(C)' = 0$, C is a constant,

2.$(x)' = 1$,

3.$(x^n)' = nx^{n-1}$.

Differentiation formulas

Suppose c is a constant and both functions $f(x)$ and $g(x)$ are differentiable, then

(a) $(cf(x))' = cf'(x)$,

(b) $(f(x) + g(x))' = f'(x) + g'(x)$,

(c) $(f(x) - g(x))' = f'(x) - g'(x)$.

Section 3-7. **Marginal analysis in business and economics.**

In economics the word *marginal* refers to a rate of change. If $C(x)$ is the total cost of producing x items, then $C'(x)$ represents the instantaneous rate of change of total cost with respect to the number of item produced.

Definition. If x is the number of units of a product produced in some time interval, then

$$\text{total cost} = C(x)$$
$$\textbf{marginal cost} = C'(x)$$
$$\text{total revenue} = R(x)$$
$$\textbf{marginal revenue} = R'(x)$$
$$\text{total profit} = P(x) = R(x) - C(x)$$
$$\textbf{marginal profit} = P'(x) = R'(x) - C'(x)$$

Note. $C(x)$ represents the total cost of producing x items. Exact cost of producing the $(x+1)$st item $= C(x+1) - C(x)$.

Marginal cost and exact cost. If $C(x)$ is the total cost producing x items, the marginal cost function approximates the exact cost of producing the $(x+1)$st item:

$$C'(x) \approx C(x+1) - C(x)$$

Similar statement can be made for $R(x)$ and $P(x)$.

Example 1. The total cost (in dollars) of producing x electric guitars is

$$C(x) = 1000 + 100x - 0.25x^2$$

(a) Find the exact cost of producing the 51st guitar.

(b) Use the marginal cost to approximate the cost of producing the 51st guitar.

Example 2. The price-demand equation and the cost function for the production of table saws are given, respectively, by

$$x = 600 - 3p \text{ and } C(x) = 7200 + 6x$$

where x is a number of saws that can be sold at a price of \$p per saw and $C(x)$ is the total cost (in dollars) of producing x saws.

(a) Express p as a function of x.

(b) Find the marginal cost.

(c) Find the revenue function.

(d) Find the marginal revenue.

(e) Find $R'(150)$ and interpret this quantity.

(f) Find the profit function in terms of x.

(g) Find the marginal profit.

(h) Find $P'(150)$ and interpret this quantity.

Marginal average cost, revenue, and profit. If x is the number of units of a product produced in some time interval, then

$$\textbf{average cost } \bar{C}(x) = \frac{C(x)}{x}$$

$$\textbf{marginal average cost} = \bar{C}'(x) = (\bar{C}(x))'$$

$$\textbf{average revenue } \bar{R}(x) = \frac{R(x)}{x}$$

$$\textbf{marginal average revenue} = \bar{R}'(x) = (\bar{R}(x))'$$

$$\textbf{average profit } \bar{P}(x) = \frac{P(x)}{x}$$

$$\textbf{marginal average profit} = \bar{P}'(x) = (\bar{P}(x))'$$

Example 3. Consider the cost function for the production of table saws from Example 2 $C(x) = 7200 + 6x$.
(a) Find $\bar{C}(x)$ and $(\bar{C}(x))'$.

(b) Find $\bar{C}(100)$ and $\bar{C}'(100)$, and interpret these quantities.

(c) Use the result in part (b) to estimate the average cost per saw at a production level of 101 saws.

Chapter 4. **Additional derivative topics.**
Section 4-1 **The constant** e **and continuous compound interest.**

The number e**.**

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.718$$

If a principal P is invested at an annual rate r (expressed as a decimal) compounded n times a year, then the amount A in the account at the end of t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

let us find

$$\lim_{n \to \infty} P\left(1 + \frac{r}{n}\right)^{nt} = Pe^{rt}$$

Continuous compound interest

$$A = Pe^{rt}$$

where
P = principal
r = annual nominal interest rate compounded continuously
t = time in years
A = amount at time t.

Example 4. Provident Bank offers a 3-years certificate of deposit (CD) that earns 5.28% compounded continuously. If \$10000 is invested in this CD, how much will it be worth in 3 years?

Example 5. How long will it take money to double if it is invested at 7% compounded continuously?

Example 6. A note will pay \$20000 at maturity 10 years from now. How much should you be willing to pay for the note now if money is worth 5.2% compounded continuously?

Section 4-2. **Derivatives of exponential and logarithmic functions.**

Let us find $(e^x)'$.

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

$$(e^x)' = e^x.$$

Example 7. Find f' for

(a) $f(x) = 3e^x + 2$

(b) $f(x) = \sqrt{x} - e^x$

(c) $f(x) = e^x + x^e$.