

Math 142, 511, 516, 517, Spring 2010

Lecture 9.

2/18/2010

Homework #10 (Section 3-5)  
Homework #11 (Section 3-7)  
are **due Thursday, Feb. 18, 11:55 PM.**

Homework #12 (Section 4-1)  
Homework #13 (Section 4-2)  
Homework #14 (Section 4-3)  
Homework #15 (Section 4-4)  
are **due Thursday, Feb. 25, 11:55 PM.**

## Table of derivatives

1.  $(C)' = 0$ ,  $C$  is a constant,

2.  $(x)' = 1$ ,

3.  $(x^n)' = nx^{n-1}$ ,

4.  $(e^x)' = e^x$ .

Section 4-2. **Derivatives of exponential and logarithmic functions.**

$$(e^x)' = e^x.$$

**Example 1.** Find the equation of the tangent line to the graph of the function

$$f(x) = x + e^x$$

at the point where  $x = 0$ .

**The derivative of  $\ln x$ .**

$$(\ln x)' = \frac{1}{x}.$$

**Example 2.** Differentiate the function  $f(x) = \ln x^2 + \sqrt[3]{x^2} - 2e^x$

$$(\log_b x)' = \frac{1}{x \ln b}.$$

$$(b^x)' = b^x \ln b$$

**Example 3.** Differentiate the following functions.

(a)  $f(x) = 3\log_2 x$       (b)  $f(x) = \log x + 5^x$

**Example 4.** The salvage value  $S$  (in dollars) of a company airplane after  $t$  years is estimated to be given by

$$S(t) = 300000(0.9)^t.$$

What is the rate of depreciation in 5 years?

### Section 4-3. **Derivatives of products and quotients.**

**Product rule.**  $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ .

**Example 5.** Find  $f'(x)$  for

(a)  $f(x) = x^3(x^2 - x + 5)$

(b)  $f(x) = (x^3 - 2x + 1)(x^2 + 1)$

(c)  $f(x) = (2x^2 + 1)e^x$ .

**Quotient rule.**  $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}.$

**Example 6.** Find  $f'$

(a)  $f(x) = \frac{x^2 + 1}{x^3 + 1}$

(b)  $f(x) = \frac{\ln x}{\ln x + 1}$

(c)  $f(x) = \frac{2^x}{e^x + x}$

## Section 4-4. The chain rule.

**Definition.** A function  $m$  is a **composite** of functions  $f$  and  $g$  if

$$m(x) = f(g(x))$$

The domain of  $m$  is the set of all numbers  $x$  such that  $x$  is in the domain of  $g$  and  $g(x)$  is in the domain of  $f$ .

**Example 7.** Let  $f(x) = x^2$  and  $g(x) = e^x$ . Find  $f(g(x))$  and  $g(f(x))$ .

**Example 8.** Write each function as a composition of two simpler functions.

(a)  $f(x) = \ln(x^2 + 3)$

(b)  $f(x) = \sqrt[3]{3 - 4x^2}$

(c)  $f(x) = (x^6 + 3x^2 - 1)^5$ .

**General power rule.** If  $u(x)$  is a differentiable function,  $n$  is any real number, then

$$([u(x)]^n)' = n[u(x)]^{n-1}u'(x).$$

**Example 9.** Find  $f'$  if

(a)  $f(x) = (2x + 3)^5$

(b)  $f(x) = \sqrt{2 - 3x^3}$

(c)  $f(x) = (\ln x + 2)^2.$

**Chain rule.** If  $m(x) = f(g(x))$ , then

$$m'(x) = f'(g(x))g'(x).$$

**Example 10.** Let  $f(x) = e^x$  and  $g(x) = x^3 + 2x - 1$ . Find

(a)  $[f(g(x))]'$

(b)  $[g(f(x))]'.$