Math 142, 511, 516, 517, Spring 2010

Lecture 10.

2/23/2010

Homework #12 (Section 4-1)
Homework #13 (Section 4-2)
Homework #14 (Section 4-3)
Homework #15 (Section 4-4)
are **due Thursday, Feb. 25, 11:55 PM.**

Quiz #5 will be held on Thursday, Feb. 25, and will cover sections 4-3 and 4-4.

Homework #16 (Section 4-7)
Homework #17 (Section 5-1)
Homework #18 (Section 5-2)
are **due Thursday, March 4, 11:55 PM.**

Table of derivatives

1. $(C)' = 0$, C is a constant,

2. $(x)' = 1$,

3. $(x^n)' = nx^{n-1}$,

4. $(e^x)' = e^x$, 4a. $(b^x)' = b^x \ln b$,

5. $(\ln x)' = \dfrac{1}{x}$, 5a. $(\log_b x)' = \dfrac{1}{x \ln b}$.

Differentiation formulas

(a) $(cf(x))' = cf'(x)$, c is a constant,

(b) $(f(x) + g(x))' = f'(x) + g'(x)$,

(c) $(f(x) - g(x))' = f'(x) - g'(x)$,

(d) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$,

(e) $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$,

(f) $(f(g(x)))' = f'(g(x))g'(x)$.

Section 4-4. **The chain rule.**

Definition. A function m is a **composite** of functions f and g if

$$m(x) = f(g(x))$$

The domain of m is the set of all numbers x such that x is in the domain of g and $g(x)$ is in the domain of f.

Chain rule. If $m(x) = f(g(x))$, then

$$m'(x) = f'(g(x))g'(x).$$

Example 1. Let $f(x) = 2^x$ and $g(x) = x^2 + 2x^3$. Find

(a) $[f(g(x))]'$ (b) $[g(f(x))]'$.

General derivative rules.

$$([u(x)]^n)' = n[u(x)]^{n-1}u'(x)$$

$$(\ln[u(x)])' = \frac{u'(x)}{u(x)}$$

$$(e^{u(x)})' = e^{u(x)}u'(x)$$

Example 2. Find f' for

(a) $f(x) = (x^3 + 2x - 1)^5$

(b) $f(x) = (x - 2\ln x)^4$

(c) $f(x) = x^2 e^{2x^3}$

(d) $f(x) = \sqrt{\ln x}$

(e) $f(x) = \dfrac{x}{(5 - 2x)^3}$

(f) $f(x) = \sqrt{\dfrac{4x + 1}{2x^2 + 1}}$

(g) $f(x) = (e^{x^2+1})^3$

(h) $f(x) = 3\ln(1 + x^2)$

Section 4-7 **Elasticity of demand.**

Question: When will a price increase lead to an increase in revenue? To answer this question we will use the notion of *elasticity of demand*.

Definition. The **relative rate of change** of a function $f(x)$ is $\frac{f'(x)}{f(x)}$. The **percentage rate of change** is $100 \times \frac{f'(x)}{f(x)}$.

Since

$$(\ln[f(x)])' = \frac{f'(x)}{f(x)},$$

the relative rate of change of $f(x)$ is the derivative of the logarithm of $f(x)$. This is also referred to as the **logarithmic derivative** of $f(x)$.

Example 3. Find the relative rate of change of $f(x) = 50x - 0.01x^2$.

Logarithmic derivatives and relative rates are used by economists to study the relationship among price changes, demand, and revenue. For most products, demand is assumed to be a decreasing function of price. That is, price increases result in lower demand, and price decreases result in higher demand.
Economists use **elasticity of demand** to study the relationship between changes in price and changes in demand.

Definition. If price and demand are related by $x = f(p)$, then the elasticity of demand is given by

$$E(p) = \frac{\text{relative rate of change of demand}}{\text{relative rate of change of price}} = -\frac{pf'(p)}{f(p)}.$$

Example 4. Given the price-demand equation

$$p + 0.005x = 30.$$

(a) Find the elasticity of demand $E(p)$.

(b) What is elasticity of demand if p=\$10? If this price is increased by 10%, what is the approximate change in demand?

(c) What is elasticity of demand if p=\$25? If this price is increased by 10%, what is the approximate change in demand?

(d) What is elasticity of demand if p=\$15? If this price is increased by 10%, what is the approximate change in demand?

If $0 < E(p) < 1$, then the demand is **inelastic**, that is the demand is not sensitive to changes in price. A change in price produces a smaller change in demand.

If $E(p) > 1$, then the demand is **elastic**, that is the demand is sensitive to changes in price. A change in price produces a larger change in demand.

If $E(p) = 1$, then the demand is **unit**. A change in price produces the same change in demand.

Now we want to see how revenue and elasticity are related.

$$\text{Revenue} = (\text{demand }) \times \ (\text{price}) = xp = pf(p).$$

$$R'(p) = f(p)(1 - E(p)).$$

Since $f(p) > 0$, $R'(p)$ and $1 - E(p)$ always have the same sign.

If $E(p) < 1$ (demand is inelastic), then $1 - E(p) > 0$ and $R'(p) > 0$.

If $E(p) > 1$ (demand is elastic), then $1 - E(p) < 0$ and $R'(p) < 0$.

Revenue and elasticity of demand.

Demand is inelastic:

A price increase will increase revenue.

A price decrease will decrease revenue.

Demand is elastic:

A price increase will decrease revenue.

A price decrease will increase revenue.

Example 5. The price-demand equation for hamburgers at a fast-food restaurant is

$$x + 400p = 2000.$$

Currently, the price of a hamburger is \$2. If the price is increased by 10%, will revenue increase or decrease?