Math 142, 511, 516, 517, Spring 2010

Lecture 11.

2/25/2010

Homework #12 (Section 4-1)
Homework #13 (Section 4-2)
Homework #14 (Section 4-3)
Homework #15 (Section 4-4)
are **due Thursday, Feb. 25, 11:55 PM.**

Homework #16 (Section 4-7)
Homework #17 (Section 5-1)
Homework #18 (Section 5-2)
are **due Thursday, March 4, 11:55 PM.**

Section 4-7 **Elasticity of demand.**

Question: When will a price increase lead to an increase in revenue?

Definition. The **relative rate of change** of a function $f(x)$ is $\frac{f'(x)}{f(x)}$. The **percentage rate of change** is $100 \times \frac{f'(x)}{f(x)}$.

Since

$$(\ln[f(x)])' = \frac{f'(x)}{f(x)},$$

the relative rate of change of $f(x)$ is the derivative of the logarithm of $f(x)$. This is also referred to as the **logarithmic derivative** of $f(x)$.

Example 1. Find the relative rate of change of $f(x) = 50x - 0.01x^2$.

For most products, demand is assumed to be a decreasing function of price.

Definition. If price and demand are related by $x = f(p)$, then the elasticity of demand is given by

$$E(p) = \frac{\text{relative rate of change of demand}}{\text{relative rate of change of price}} = -\frac{pf'(p)}{f(p)}.$$

Example 2. Given the price-demand equation

$$p + 0.005x = 30.$$

(a) Find the elasticity of demand $E(p)$.

(b) What is elasticity of demand if p=\$10? If this price is increased by 10%, what is the approximate change in demand?

(c) What is elasticity of demand if p=\$25? If this price is increased by 10%, what is the approximate change in demand?

(d) What is elasticity of demand if p=\$15? If this price is increased by 10%, what is the approximate change in demand?

If $0 < E(p) < 1$, then the demand is **inelastic**, that is the demand is not sensitive to changes in price. A change in price produces a smaller change in demand.

If $E(p) > 1$, then the demand is **elastic**, that is the demand is sensitive to changes in price. A change in price produces a larger change in demand.

If $E(p) = 1$, then the demand is **unit**. A change in price produces the same change in demand.

Now we want to see how revenue and elasticity are related.

$$\text{Revenue} = (\text{demand }) \times \; (\text{price}) = xp = pf(p).$$

$$R'(p) = f(p)(1 - E(p)).$$

Since $f(p) > 0$, $R'(p)$ and $1 - E(p)$ always have the same sign.

If $E(p) < 1$ (demand is inelastic), then $1 - E(p) > 0$ and $R'(p) > 0$.

If $E(p) > 1$ (demand is elastic), then $1 - E(p) < 0$ and $R'(p) < 0$.

Revenue and elasticity of demand.

Demand is inelastic:

A price increase will increase revenue.

A price decrease will decrease revenue.

Demand is elastic:

A price increase will decrease revenue.

A price decrease will increase revenue.

Example 3. The price-demand equation for hamburgers at a fast-food restaurant is

$$x + 400p = 2000.$$

Currently, the price of a hamburger is \$2. If the price is increased by 10%, will revenue increase or decrease?

Chapter 5. **Graphing and optimization.**
Section 5-1. **First derivative and graphs.**

Definition. The values of x in the domain of f where $f'(x) = 0$ or where $f'(x)$ does not exist are called the **critical values** of f.

Example 4. Find all critical values of the function $f(x) = x\sqrt{x^2 + 1}$.

Definition. The function f is **increasing** on an interval (a, b) if

$$f(x_2) > f(x_1)$$

whenever $a < x_1 < x_2 < b$, and f is **decreasing** on (a, b) if

$$f(x_2) < f(x_1)$$

whenever $a < x_1 < x_2 < b$.

Increasing / decreasing test
(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval
(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval

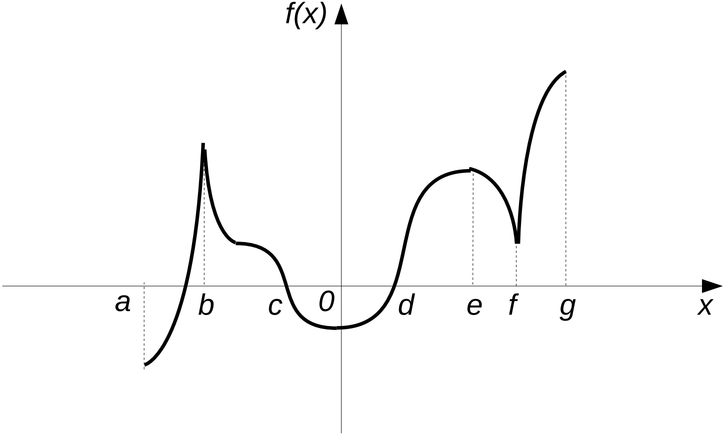
To find intervals on which a function is increasing or decreasing, we will construct a sign chart for $f'(x)$ to determine which values of x make $f'(x) > 0$ and which values make $f'(x) < 0$.

Example 5. Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for

$$f(x) = x^4 - 18x^2.$$

Definition. A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. [This means that $f(c) \geq f(x)$ for all x in some *open* interval containing c]. Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c. The quantity $f(c)$ is called a **local extremum** if it either a local maximum or a local minimum.

Example 5. Given the graph of the function f.



(a) Identify intervals on which $f(x)$ is increasing. Is decreasing.

(b) Identify the x coordinates of the points where $f(x)$ has a local maximum. A local minimum.

Fermat's theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$

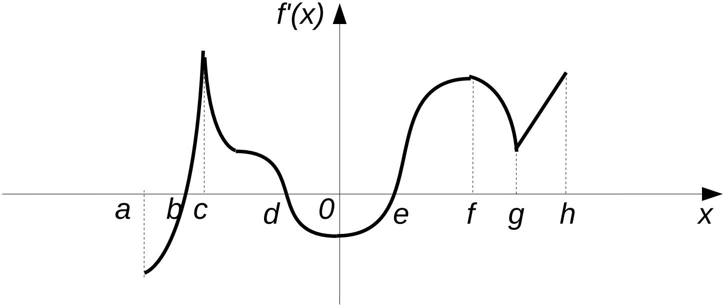
The first derivative test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local max at c.

(b) If f' changes from negative to positive at c, then f has a local min at c.

(c) If f' does not change sign c, then f has a no local max or min at c.

Example 6. Given the graph of $f'(x)$.



(a) Identify intervals on which $f(x)$ is increasing. Is decreasing.

(b) Identify the x coordinates of the points where $f(x)$ has a local maximum. A local minimum.

Example 7. Find the critical values, the intervals on which $f(x)$ is increasing, decreasing, and the local extrema.

Theorem. If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, $a_n \neq 0$, is an n-th degree polynomial, then f has at most n x intercepts and at most $n - 1$ local extrema.