Math 142, 511, 516, 517, Spring 2010

Lecture 12.

3/2/2010

Homework #16 (Section 4-7) is **due Thursday, March 4, 11:55 PM.**

The due date for the Homework #17 (Section 5-1) and the Homework #18 (Section 5-2) **is moved to Monday, March 8, 11:55 PM.**

Quiz 6 will be held on Thursday, March 4. It will cover Sections 4-7 and 5-1.

Section 5-1. **First derivative and graphs.**

Definition. The values of x in the domain of f where $f'(x) = 0$ or where $f'(x)$ does not exist are called the **critical values** of f.

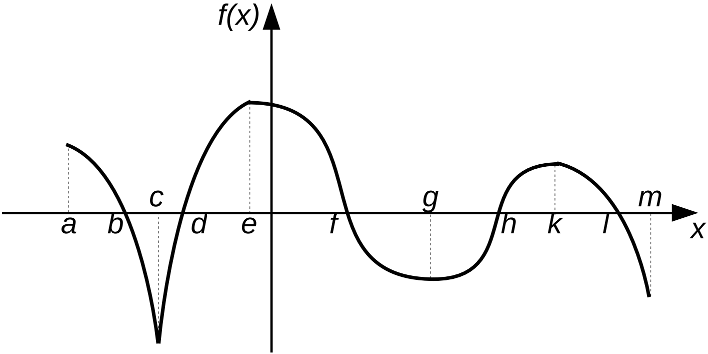
Definition. The function f is **increasing** on an interval (a, b) if $f(x_2) > f(x_1)$ whenever $a < x_1 < x_2 < b$, and f is **decreasing** on (a, b) if $f(x_2) < f(x_1)$ whenever $a < x_1 < x_2 < b$.

Increasing / decreasing test
(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval
(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval

To find intervals on which a function is increasing or decreasing, we will construct a sign chart for $f'(x)$ to determine which values of x make $f'(x) > 0$ and which values make $f'(x) < 0$.

Definition. A function f has a **local maximum** at c if $f(c) \geq f(x)$ when x is near c. Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c. The quantity $f(c)$ is called a **local extremum** if it either a local maximum or a local minimum.

Example 1. Given the graph of the function f.



(a) What are the x-coordinate(s) of the points where $f'(x)$ does not exist?

(b) Identify intervals on which $f(x)$ is increasing. Is decreasing.

(c) Identify the x coordinates of the points where $f(x)$ has a local maximum. A local minimum.

Fermat's theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$

The first derivative test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local max at c.

(b) If f' changes from negative to positive at c, then f has a local min at c.

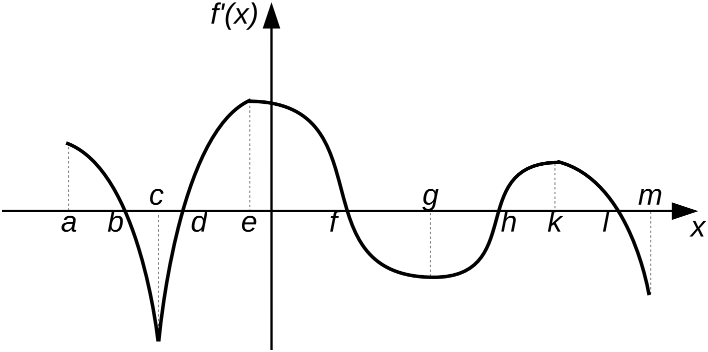
(c) If f' does not change sign at c, then f has a no local max or min at c.

Example 2. Find the critical values of f, the intervals on which f is increasing, the intervals on which f is decreasing, and the local extrema for

$$f(x) = 2\sqrt[3]{x^2 - 4}$$

Do not graph.

Example 3. Given the graph of $f'(x)$.



(a) Identify intervals on which f is increasing. Is decreasing.

(b) Identify the x coordinates of the points where f has a local maximum. A local minimum.

Section 5-2. **Second derivative and graphs.**

Definition. The graph of the function f is **concave upward (CU)** on the interval (a, b) if $f'(x)$ is increasing on (a, b) and is **concave downward (CD)** on the interval (a, b) if $f'(x)$ is decreasing on (a, b)

Definition. For $y = f(x)$, the **second derivative** of f, provided that it exists, is

$$f''(x) = \frac{d}{dx} f'(x)$$

Other notation for $f''(x)$ are

$$\frac{d^2y}{dx^2} \quad y''$$

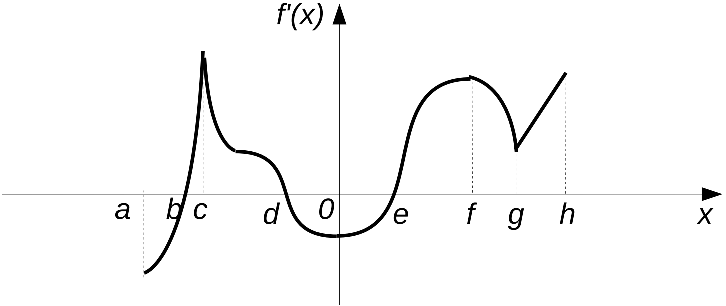
Example 4. Find f'' for

(a) $f(x) = -6x^{-2} + 12x^{-3}$, (b) $f(x) = xe^x$,

(c) $f(x) = 2\sqrt[3]{x^2 - 4}$.

Definition. An **inflection point** is a point on the graph of the function where the concavity changes.

Example 5. Given the graph of $f'(x)$.



(a) Identify intervals on which f is concave upward. Concave downward.

(b) Find the x-coordinates of inflection points.

Concavity test

(a) If $f''(x) > 0$ on an interval, then f is CU on this interval.

(b) If $f''(x) < 0$ on an interval, then f is CD on this interval.

The second derivative test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local min at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local max at c.

If $y = f(x)$ is continuous on (a, b) and has an inflection point at $x = c$, then either $f''(c) = 0$ or $f''(c)$ does not exist.

Example 5. Find inflection points and the intervals of which the graph of $f(x) = x^4 - 2x^3 - 36x + 12$ is CU.