

Math 142, 511, 516, 517, Spring 2010

Lecture 14.

3/9/2010

Homework #19 (Section 5-4)

Homework #20 (Section 5-5)

are due Thursday, March 11, 11:55 PM.

Quiz 7 will be held on Thursday, March 11. It will cover sections 5-2, 5-4, 5-5.

Section 5-5 **Absolute maxima and minima.**

Definition. If $f(c) \geq f(x)$ for all x in the domain of f , then $f(c)$ is called the **absolute maximum value** of f .

If $f(c) \leq f(x)$ for all x in the domain of f , then $f(c)$ is called the **absolute minimum value** of f .

The absolute min and the absolute max are referred to as **absolute extrema**.

Extreme value theorem. A function f is continuous on a closed interval $[a, b]$ has both an absolute maximum value and an absolute minimum value on that interval.

Locating absolute extrema. Absolute extrema (if they exist) must always occur at critical values or at endpoints.

Finding absolute extrema on a closed interval.

- Step 1. Check to make certain that f is continuous over $[a, b]$.
- Step 2. Find the critical values in the interval (a, b) .
- Step 3. Evaluate f at the endpoints a and b and at the critical values found in Step 2.
- Step 4. The absolute maximum $f(x)$ on $[a, b]$ is the largest value found in Step 3.
- Step 5. The absolute minimum $f(x)$ on $[a, b]$ is the smallest value found in Step 3.

Example 1. Find the absolute maximum and minimum, if either exists, for the function $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval
(a) $[-1, 5]$ (b) $[-1, 3]$ (c) $[2, 5]$.

The second derivative test Suppose f'' is continuous near c .

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has an absolute min at c .

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has an absolute max at c .

Note. The second derivative test cannot be applied when $f''(c) = 0$ or $f''(c)$ does not exist.

Example 2. Assuming that the function $f(x)$ is continuous on the interval $(-\infty, \infty)$, indicate whether each of the points listed below is a local maximum, local minimum or neither. If this cannot be determined from the stated information, then indicate this in your answer.

(a) $(2, f(2))$ if $f'(2) = 0$ and $f''(2) > 0$

(b) $(-1, f(-1))$ if $f'(-1) = 0$ and $f''(-1) < 0$

(c) $(5, f(5))$ if $f'(5) = 0$ and $f''(5)$ does not exist.

(d) $(6, f(6))$ if $f'(6) = 1$ and $f''(6) > 0$.

(e) $(2, f(2))$ if $f'(2) = 0$ and $f''(2) = 0$

Example 3. Find the absolute minimum value on $(0, \infty)$ for the function $f(x) = 4 + x + \frac{9}{x}$.

Example 4. Find the absolute maximum and minimum, if either exists, for the following functions.

(a) $f(x) = -x^2 - 6x + 9$

(b) $f(x) = \frac{-8x}{x^2 + 4}$.