

Math 141, 511, 516, 517, Spring 2010

Lecture 16

03/30/2010

The Math 142 help session on Thursday, Apr. 1, is cancelled.

Homework #22 (Section 6-1)

Homework #23 (Section 6-2)

Homework #24 (Section 6-4)

are **due Thursday, April 8, 11:55 PM.**

Chapter 6. Integration

Section 6-1 Antiderivatives and indefinite integrals.

Definition Function $F(x)$ is called an **antiderivative** of $f(x)$ if $F'(x) = f(x)$.

Theorem If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

$$F(x) + C$$

where C is a constant.

Example 1. Find the general antiderivative of the function

$$f(t) = \frac{t^2 + 3t + 2}{t}$$

Indefinite integrals: formulas and properties.

The symbol

$$\int f(x)dx$$

is called the **indefinite integral** and represents the family of all antiderivatives of $f(x)$.

$$\int f(x)dx = F(x) + C \quad \text{if} \quad F'(x) = f(x)$$

The symbol \int is called an **integral sign**, $f(x)$ is called the **integrand**. The symbol dx indicates that the antidifferentiation is performed with respect to the variable x . C is called the **constant of integration**.

The procedure of evaluating an integral is called **integration**.

Table of indefinite integrals.

1. $\int a dx = ax + C$, a is a constant

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x dx = e^x + C$

Properties of indefinite integrals.

For k a constant,

1. $\int kf(x)dx = k \int f(x)dx$

2. $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

Example 2. Find the indefinite integral.

(a) $\int t^2 dt$ (b) $\int (x^2 - x + 4) dx$ (c) $\int x^2(x^3 - 2x + 1) dx$

(d) $\int (x^2 - 3)(1 + x^3) dx$ (e) $\int \frac{5}{\sqrt[3]{t}} dt$ (f) $\int \frac{1 - y^2}{y^3} dy$

(g) $\int \left(\frac{2}{\sqrt[4]{x}} - \sqrt[5]{x^3} \right) dx$ (h) $\int \frac{4e^x - 3x^2}{2} dx$

Example 3. Find the equation of the curve that passes through $(-1, 6)$ if the slope of the curve is given by $\frac{dy}{dx} = 4x^3 + 2x$ at any point x .

Example 4. The marginal average cost of producing x sports watches is given by

$$\bar{C}'(x) = -\frac{1000}{x^2}, \quad \bar{C}(100) = 25$$

where $\bar{C}(x)$ is the average cost in dollars. Find the average cost function and the cost function.

Section 6-2. Integration by substituting.

Reversing the chain rule.

$$\int f'(g(x))g'(x)dx = f(g(x)) + C$$

General indefinite integral formulas.

$$1. \int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C, \quad n \neq -1.$$

$$2. \int e^{f(x)} f'(x)dx = e^{f(x)} + C$$

$$3. \int \frac{1}{f(x)} f'(x)dx = \ln |f(x)| + C.$$

Example 5. Find each indefinite integral.

$$(a) \int (x^3 + 4)^{-2} x^2 dx \quad (b) \int x e^{x^2} dx \quad (c) \int \frac{x}{2 - x^2} dx$$