Math 141, 511, 516, 517, Spring 2010

Lecture 17

04/01/2010

The Math 142 help session on Thursday, Apr. 1, is cancelled.

The Math 142 week in review for this Sunday will only be available online (not live) because of the Easter holiday.

Homework #22 (Section 6-1)
Homework #23 (Section 6-2)
Homework #24 (Section 6-4)
are **due Thursday, April 8, 11:55 PM.**

Table of indefinite integrals.

1. $\int a\ dx = ax + C$, a is a constant

2. $\displaystyle\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

3. $\displaystyle\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x dx = e^x + C$

Properties of indefinite integrals.

For k a constant,

1. $\int kf(x)dx = k\int f(x)dx$

2. $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

Section 6-2. **Integration by substituting.**

Reversing the chain rule.

$$\int f'(g(x))g'(x)dx = f(g(x)) + C$$

General indefinite integral formulas.

1. $\displaystyle\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C, \quad n \neq -1.$

2. $\int e^{f(x)} f'(x)dx = e^{f(x)} + C$

3. $\displaystyle\int \frac{1}{f(x)} f'(x)dx = \ln|f(x)| + C.$

Example 1. Find each integral

(a) $\displaystyle\int \frac{1}{x^2} e^{-1/x} dx$ (b) $\displaystyle\int \frac{x^3}{\sqrt{2x^4+3}} dx$ (c) $\displaystyle\int \frac{(\ln x)^3}{x} dx$

Integration by substitution.

Definition. If $y = f(x)$ defines a differentiable function, then

1. The **differential** dx of the independent variable x is an arbitrary real number.
2. The **differential** dy of the dependent variable y is defined as

$$dy = f'(x)dx.$$

Integration by substitution.

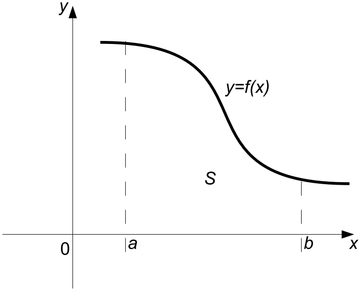
1. Select a substitution that appears to simplify the integrand.
2. Express the integrand entirely in terms of u and du, completely eliminating the original variable and its differential.
3. Evaluate the new integral if possible.
4. Express the antiderivative found in previous step in terms of original variable.

Example 2. Find each integral.

(a) $\int (5x-3)^{10} dx$ (b) $\displaystyle\int \frac{1}{2x-3} dx$ (c) $\int x\sqrt{x+4}\,dx$

Section 6-4. **The definite integral.**

Problem Find the area of the region S that lies under the curve $y = f(x)$ from a to b.



$$S = \{(x, y) : \ a \le x \le b, \ 0 \le y \le \ f(x)\}$$

We partition $[a, b]$ into n subintervals of equal length $\Delta x = (b - a)/n$ with endpoints

$$a = x_0 < x_1 < x_2 < ... < x_n = b.$$

Then, we have

Left sum $L_n = f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x = \sum_{k=1}^{n} f(x_{k-1})\Delta x$

Right sum $R_n = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x = \sum_{k=1}^{n} f(x_k)\Delta x$

Midpoint sum $M_n = f(\bar{x}_0)\Delta x + f(\bar{x}_1)\Delta x + ... + f(\bar{x}_n)\Delta x = \sum_{k=1}^{n} f(\bar{x}_k)\Delta x$

here $\bar{x}_k$ is the midpoint of the subinterval $[x_{k-1}, x_k]$ Then we can approximate the area of the region S using one of these sums.
Theorem. If $f(x) > 0$ and either increasing on $[a, b]$ or decreasing on $[a, b]$, then its left, right, and midpoint sums approach the same real number as $n \to \infty$.

Example 3. Approximate the area under the curve $y = x^2 + 3x - 2$ from 1 to 4 using

1. the right sum R_6
2. the left sum L_6
3. the midpoint sum M_6.