Math 141, 511, 516, 517, Spring 2010

Lecture 18

04/06/2010

Homework #22 (Section 6-1)
Homework #23 (Section 6-2)
Homework #24 (Section 6-4)
are **due Thursday, April 8, 11:55 PM.**

Quiz 9 is due Thursday, April 8.

Quiz 10 will be held in class on Thursday, April 8. It will cover sections 6-4 and 6-5.

Section 6-4. **The definite integral.**

We partition $[a, b]$ into n subintervals of equal length $\Delta x = (b - a)/n$ with endpoints

$$a = x_0 < x_1 < x_2 < ... < x_n = b.$$

Then, we have

Left sum $L_n = f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x = \sum_{k=1}^{n} f(x_{k-1})\Delta x$

Right sum $R_n = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x = \sum_{k=1}^{n} f(x_k)\Delta x$

Riemann sum $S_n = f(c_1)\Delta x + f(c_2)\Delta x + ... + f(\bar{x}_n)\Delta x = \sum_{k=1}^{n} f(c_k)\Delta x$

here c_k belongs to the subinterval $[x_{k-1}, x_k]$.
Theorem. If f is continuous function on $[a, b]$, then the Riemann sums approach to a real number limit I as $n \to \infty$.

Definition of a definite integral. Let f be continuous function on $[a, b]$. Then the limit I of Riemann sums for f on $[a, b]$ is called the **definite integral** of f from a to b and is denoted by

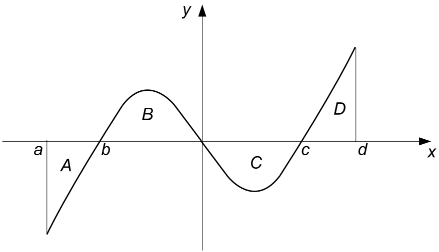
$$\int_a^b f(x)dx$$

The **integrand** is $f(x)$, the **lower limit of integration** is a, and the **upper limit of integration** is b.

Example 1. Partition interval [0,4] into four subintervals of equal length. Calculate the Riemann sum S_4 for the function $f(x) = x^2 - 4x + 6$ if $c_1 = .4$, $c_2 = 1.2$, $c_3 = 2.3$, and $c_4 = 3.6$.

Because area is a positive quantity, the definite integral represents the cumulative sum of the signed areas between the graph of f and the x-axis from $x = a$ to $x = b$, where the areas above the x-axis are counted positively and the areas below the x-axis are counted negatively.

Example 2. Calculate definite integrals by reffering to the figure



if area $A = 1.22$, area $B = 2.3$, area $C = 2.5$, and area $D = 1.6$.

(a) $\int\limits_a^b f(x)dx$, (b) $\int\limits_a^c f(x)dx$, (c) $\int\limits_b^d f(x)dx$

Properties of the definite integral

1. $\int\limits_a^a f(x)dx = 0$
2. $\int\limits_a^b f(x)dx = -\int\limits_b^a f(x)dx$
3. $\int\limits_a^b cdx = c(b-a)$, where c is a constant
4. $\int\limits_a^b cf(x)dx = c\int\limits_a^b f(x)dx$, where c is a constant
5. $\int\limits_a^b [f(x) \pm g(x)]dx = \int\limits_a^b f(x)dx \pm \int\limits_a^b g(x)dx$
6. $\int\limits_a^b f(x)dx = \int\limits_a^c f(x)dx + \int\limits_c^b f(x)dx$, where $a < c < b$

Example 3. Given that $\int\limits_1^4 x dx = \frac{15}{2}$, $\int\limits_1^4 x^2 dx = 21$, and $\int\limits_4^6 x^2 dx = \frac{152}{3}$.

Find

(a) $\int\limits_1^1 (2x - 3x^2) dx$ (b) $\int\limits_1^6 x^2 dx$ (c) $\int\limits_1^4 x(x-1) dx$

Section 6-5. **The fundamental theorem of calculus.**

Theorem. If f is continuous function on $[a, b]$, and F is any antiderivative of f, then

$$\int\limits_a^b f(x) dx = F(b) - F(a)$$

Example 4. Evaluate the following definite integrals:

(a) $\int_1^2 \left(e^x - \frac{2}{x^3}\right) dx$ (b) $\int\limits_0^1 \frac{x-1}{x^2 - 2x + 3} dx$ (c) $\int\limits_0^2 \frac{x}{\sqrt{4-x}} dx$.

Example 5. Evaluate the definite integral

$$\int_{1.7}^{3.5} x \ln x dx$$

to three decimal places.

Example 6. A managerial service determines that the rate of increase in maintenance costs (in dollars per year) for a particular apartment complex is given approximately by

$$M'(x) = f(x) = 90x^2 + 5000$$

where x is the age of the apartment complex in years and $M(x)$ is the total (accumulated) cost of maintenance for x years. Write a definite integral that will give the total maintenance costs from the end of the second year to the end of the seventh year after the apartment complex was built, and evaluate the integral.

Average value of a continuous function f **over** $[a, b]$ is

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)dx$$

Example 7. Find the average value of the function $f(x) = \sqrt[3]{x}$ over the interval $[1, 8]$.