

Math 141, 511, 516, 517, Spring 2010

Lecture 20

04/13/2010

Homework #25 (Section 6-5)

Homework #26 (Section 7-1)

Homework #27 (Section 7-2)

are **due Thursday, April 15, 11:55 PM.**

Quiz 11 will be held on Thursday, April 15. It will cover sections 7-1, 7-2, 8-1.

Section 7-2. Applications in business and economics.

Consumers' and producers' surplus.

Definition. Consumers' Surplus If (\bar{x}, \bar{p}) is a point on the graph of the price-demand equation $p = D(x)$ for a particular product, then the **consumers' surplus CS** at a price level of \bar{p} is

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}] dx$$

which is the area between $p = \bar{p}$ and $p = D(x)$ from $x = 0$ to $x = \bar{x}$.

The consumers' surplus represents the total savings to consumers who are willing to pay more than \bar{p} for the product, but are still able to buy the product for \bar{p} .

Definition. Producers' surplus.

If (\bar{x}, \bar{p}) is a point on the graph of the price-supply equation $p = S(x)$, then the **producers' surplus PS** at a price level of \bar{p} is

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)] dx$$

which is the area between $p = \bar{p}$ and $p = S(x)$ from $x = 0$ to $x = \bar{x}$.

The producers' surplus represents the total gain to producers who are willing to supply units at a lower price than \bar{p} but are still able to supply units at \bar{p} .

If $p = D(x)$ and $p = S(x)$ are the price-demand and price-supply equations, respectively, for a product and if (\bar{x}, \bar{p}) is the point of intersection of these equations, then \bar{p} is called the **equilibrium price** and \bar{x} is called the **equilibrium quantity**. If the price stabilizes at the equilibrium price \bar{p} , then this is the price level that will determine both the consumers' surplus and the producers' surplus.

Example 1. Find the equilibrium price and then find the consumers' surplus and producers' surplus at the equilibrium price level if

$$p = D(x) = 50 - 0.1x, \quad p = S(x) = 11 + 0.05x.$$

Chapter 8. **Multivariable calculus.**
Section 8-1. **Functions of several variables.**

An equation of the form

$$z = f(x, y)$$

describes a **function of two variables** if, for each permissible ordered pair (x, y) , there is one and only one value of z determined by $f(x, y)$. The variables x and y are **independent variables**, and the variable z is a **dependent variable**. The set of all ordered pairs of permissible values of x and y is the **domain** of the function, and the set of all corresponding values $f(x, y)$ is the **range** of the function. We will assume that the domain of a function $z = f(x, y)$ is the set of all ordered pairs of real numbers (x, y) such that $f(x, y)$ is also a real number.

Similarly we can define functions of three and more variables.

Example 2. Let $f(x, y) = 2x - 3y + 4$. Find $f(4, -2)$.

Example 3. Let $A(P, r, t, n) = P \left(1 + \frac{r}{n}\right)^{tn}$. Find $A(10, 0.04, 3, 2)$.

Example 4. A company manufactures 10- and 13- speed bicycles. The weekly demand and cost equations are

$$p = 230 - 9x + y$$

$$q = 130 + x - 4y$$

$$C(x, y) = 200 + 80x + 30y$$

where $\$p$ is the price of a 10-speed bicycle, $\$q$ is the price of a 13-speed bicycle, x is the weekly demand for 10-speed bicycle, y is the weekly demand for 13-speed bicycle, $C(x, y)$ is the cost function. Find the weekly revenue function $R(x, y)$ and the weekly profit function $P(x, y)$. Evaluate $R(10, 15)$ and $P(10, 15)$.

Cobb-Douglas production function

$$f(x, y) = kx^m y^n$$

where k , m , and n are positive constants with $m + n = 1$. Economists use this function to describe the number of units $f(x, y)$ produced from the utilization of x units of labor and y units of capital (for equipment such as tools, machinery, buildings and so on).

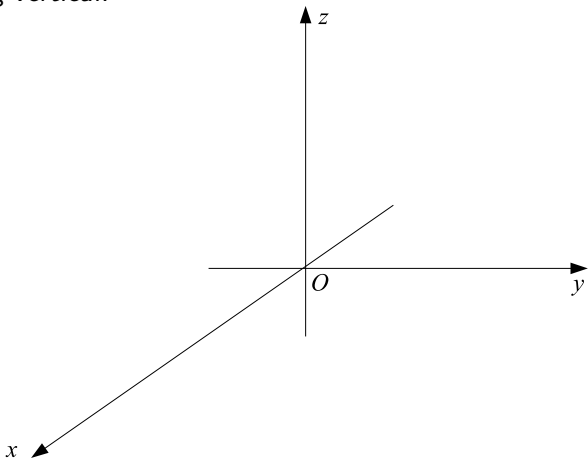
Example 5. The Cobb-Douglas production function for a petroleum company is given by

$$f(x, y) = 10x^{0.4}y^{0.6}$$

where x is the utilization of labor and y is the utilization of capital. If the company uses 1250 units of labor and 1700 units of capital, how many units of petroleum will be produced?

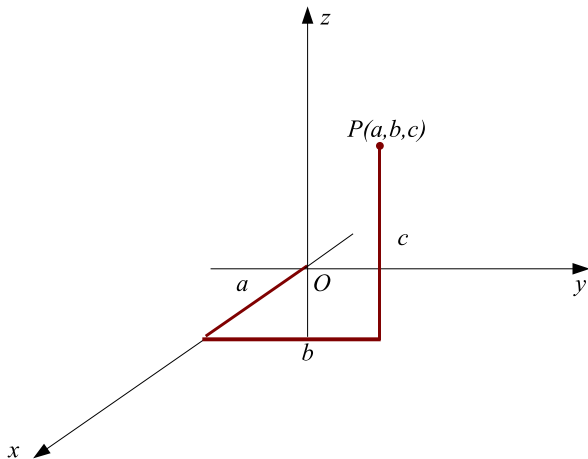
Three-dimensional coordinate system.

We first choose a fixed point O (the origin) and three directed lines through O that are perpendicular to each other, called the **coordinate axes** and labeled the x -axis, y -axis, and z -axis. Usually we think of the x and y -axes as being horizontal and z -axis as being vertical.



The three coordinate axes determine the three **coordinate planes**. The xy -plane contains the x - and y -axes and its equation is $z = 0$, the xz -plane contains the x - and z -axes and its equation is $y = 0$, The yz -plane contains the y - and z -axes and its equation is $x = 0$. These three coordinate planes divide space into eight parts called **octants**. The **first octant** is determined by positive axes.

Take a point P in space, let a be directed distance from yz -plane to P , b be directed distance from xz -plane to P , and c be directed distance from xy -plane to P .



We represent the point P by the ordered triple (a, b, c) of real numbers, and we call a , b , and c the **coordinates** of P .

Example 6. Let $f(x, y) = 100 + 10x + 25y - x^2 - 5y^2$. Describe the cross-sections of the surface $z = f(x, y)$ produced by cutting it with the planes $x = 1$ and $y = 3$.