

Math 141, 511, 516, 517, Spring 2010

Lecture 21

04/15/2010

Homework #25 (Section 6-5)

Homework #26 (Section 7-1)

Homework #27 (Section 7-2)

are **due Thursday, April 15, 11:55 PM.**

Homework #28 (Section 8-1)

Homework #29 (Section 8-2)

Homework #30 (Section 8-3)

are **due Thursday, April 22, 11:55 PM.**

**Quiz 12** is due Thursday, April 22.

**Test 3** will be held on Thursday, April 22. It will cover sections 6-1, 6-2, 6-4, 6-5, 7-1, 7-2 (Topic Consumers' and Producers' Surplus), 8-1 – 8-3.

## Section 8-2. Partial derivatives.

If  $z = f(x, y)$ , then the **partial derivative of  $f$  with respect to  $x$** , is defined by

$$\frac{\partial z}{\partial x} = f_x = f_x(x, y) = \lim_{h \rightarrow 0} \frac{f(x + h, y) - f(x, y)}{h}$$

the **partial derivative of  $f$  with respect to  $y$** , is defined by

$$\frac{\partial z}{\partial y} = f_y = f_y(x, y) = \lim_{k \rightarrow 0} \frac{f(x, y + k) - f(x, y)}{k}$$

**Example 1.** Find the indicated partial derivative for the following functions.

1.  $f_x(x, y)$  if  $f(x, y) = 4x - 3y + 5$
2.  $f_y(x, y)$  if  $f(x, y) = x^2 - 3xy - 4y^2$
3.  $f_x$  if  $f(x, y) = (5x + 2y)^8$

**Example 2.** A company spends \$ $x$  per week on newspapers advertising and \$ $y$  per week on televisions advertising. Its weekly sales were found to be given by

$$S(x, y) = 10x^{0.2}y^{0.8}$$

Find  $S_x(3000, 2000)$ .

**Second-order partial derivatives.** If  $z = f(x, y)$ , then

$$f_{xx} = f_{xx}(x, y) = \frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial x} \right)$$

$$f_{yy} = f_{yy}(x, y) = \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial y} \right)$$

$$f_{xy} = f_{xy}(x, y) = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} \right)$$

**Example 3.** Find  $f_x$ ,  $f_y$ ,  $f_{xx}$ ,  $f_{yy}$  and  $f_{xy}$  for the function

$$f(x, y) = x^3 \ln y + 4y^2 e^x$$