Math 141, 511, 516, 517, Spring 2010

Lecture 22

04/20/2010

Homework #28 (Section 8-1)
Homework #29 (Section 8-2)
Homework #30 (Section 8-3)
are **due Thursday, April 22, 11:55 PM.**

Quiz 12 is due Thursday, April 22.

Test 3 will be held on Thursday, April 22. It will cover sections 6-1, 6-2, 6-4, 6-5, 7-1, 7-2 (Topic Consumers' and Producers' Surplus), 8-1 – 8-3.

The last day when you can make up quizzes or tests is Tuesday, May 4.

Section 8-3. **Maxima and minima**

We say that $f(a, b)$ **is a local maximum** if there exists a circular region in the domain of f with (a, b) as the center, such that

$$f(a, b) \geq f(x, y)$$

for all (x, y) in the region. Similarly, we say that $f(a, b)$ **is a local minimum** if there exists a circular region in the domain of f with (a, b) as the center, such that

$$f(a, b) \leq f(x, y)$$

for all (x, y) in the region.

Theorem. Let $f(a, b)$ be a local extremum for the function f. If both f_x and f_y exist at (a, b), then

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0$$

Theorem gives us *necessary* (but not *sufficient*) conditions for $f(a, b)$ to be a local extremum. We thus find all points (a, b) such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ and test further to determine whether $f(a, b)$ is a local extremum or a saddle point.

Second-derivative test for local extrema. For $z = f(x, y)$ if
1. $f_x(a, b) = 0$ and $f_y(a, b) = 0$
2. All second-order partial derivatives of f exist in some circular region containing (a, b) as center.
3. $A = f_{xx}(a, b)$, $B = f_{xy}(a, b)$, $C = f_{yy}(a, b)$

Then

Case 1. If $AC - B^2 > 0$ and $A < 0$, then $f(a, b)$ is a local maximum.

Case 2. If $AC - B^2 > 0$ and $A > 0$, then $f(a, b)$ is a local minimum.

Case 3. If $AC - B^2 < 0$, then f has a saddle point at (a, b).

Case 4. If $AC - B^2 = 0$, the test fails.

Example Find local extrema for the function

$$f(x, y) = 2x^2 - xy + y^2 - x - 5y + 8$$