

Answers for the sample problems for the final exam

1. $p(x) = x^2 - 2x + 3$

2. (a) $\det(A) = 1$.

(b) $A^{-1} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{pmatrix}$.

3. $T = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 2 \\ -1 & 1 & -3 \end{pmatrix}$

4. (a) $A = \begin{pmatrix} 2 & 4 & -2 \\ 2 & 4 & -2 \\ 1 & 2 & -1 \end{pmatrix}$

(b) Range of L is spanned by the vector $(1, 2, -1)$, kernel of L is spanned by the vectors $(-2, 1, 0)$, $(1, 0, 1)$.

5. (a) 2

(b) $2\sqrt{3}$

6. (a) $\lambda_1 = 1$, $\lambda_2 = -1$, $\lambda_3 = 3$.

(b) $x_1 = (2, 1, 0)$, $x_2 = (0, 1, 0)$, $x_3 = (4, 3, 8)$.

(c) $u_1 = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$, $u_2 = \left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right)$, $u_3 = (0, 0, 1)$.

(d) $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, $X = \begin{pmatrix} 2 & 0 & 4 \\ 1 & 1 & 3 \\ 0 & 0 & 8 \end{pmatrix}$.

7. $x(t) = e^{4t}$, $y(t) = e^{4t}$.