MATH 304–505 Spring 2009

Answers for the sample problems for Test 1

1. $(-1, \frac{4}{3}, \frac{2}{3})$

2. $\det(A) = -1$.

3. $A^{-1} = \begin{pmatrix} 3/14 & -3/7 & 1/7 \\ 1/7 & 5/7 & -4/7 \\ -1/14 & 1/7 & 2/7 \end{pmatrix}$

4. (a) yes

   (b) no

   (c) yes

   (d) no

5. (a) rank of B = 3
   nullity of B = 4 − rank of B = 4 − 3 = 1

   (b) Vectors $\mathbf{v}_1 = (1, 1, 2, -1)$, $\mathbf{v}_2 = (0, 1, -4, -1)$, and $\mathbf{v}_3 = (0, 0, 1, 0)$ form the basis for the row space for B.

   One of the vectors $\mathbf{e}_1 = (1, 0, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0, 0)$, $\mathbf{e}_4 = (0, 0, 0, 1)$ can be chosen to extend the basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ to a basis of $\mathbb{R}^4$.

6. Since $W[f_1, f_2, f_3](x) = 2x^2 + 3x + 2$, and it is never zero on $\mathbb{R}$, functions $f_1(x)$, $f_2(x)$ and $f_3(x)$ are linearly independent.

7. Assume the contrary: neither of the subspaces V and W is contained in the other. Then there exist vectors $\mathbf{v} \in V$ and $\mathbf{w} \in W$ such that $\mathbf{v} \notin W$ and $\mathbf{w} \notin V$. Let $\mathbf{x} = \mathbf{v} + \mathbf{w}$. Since $\mathbf{v}, \mathbf{w} \in V \cup W$ and $V \cup W$ is a subspace, it follows that $\mathbf{x} \in V \cup W$. That is, $\mathbf{x} \in V$ or $\mathbf{x} \in W$. However in the case $\mathbf{x} \in V$ we have $\mathbf{w} = \mathbf{x} - \mathbf{v} \in V$, while in the case $\mathbf{x} \in W$ we have $\mathbf{v} = \mathbf{x} - \mathbf{w} \in W$. In either case we arrive at a contradiction. Thus the initial assumption was wrong. That is, one of the subspaces V and W does contain the other.