MATH 304–505 Spring 2009

Sample problems for Test 1

1. Find the point of intersection of the planes $x+2y-z=1$, $x-3y=-5$, and $2x+y+z=0$ in $\mathbb{R}^3$.

2. Let $A = \begin{pmatrix} 1 & -2 & 4 & 1 \\ 2 & 3 & 2 & 0 \\ 2 & 0 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix}$. Evaluate the determinant of the matrix A.

3. Let $A = \begin{pmatrix} 4 & 2 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{pmatrix}$. Find the inverse matrix A^{-1}.

4. Let $\mathcal{P}_4$ be the vector space of all polynomials (with real coefficients) of degree less than 4. Determine which of the following subsets of $\mathcal{P}_4$ are vector subspaces. Briefly explain.

   (a) The set S_1 of polynomials $p(x) \in \mathcal{P}_4$ such that $p(0) = 0$.
   (b) The set S_2 of polynomials $p(x) \in \mathcal{P}_4$ such that $p(0)p(1) = 0$.
   (c) The set S_3 of polynomials $p(x) \in \mathcal{P}_4$ such that $(p(0))^2 + (p(1))^2 = 0$.
   (d) The set S_4 of polynomials $p(x) \in \mathcal{P}_4$ such that $p(0) = 0$ and $p(1) = 1$.

5. Let $B = \begin{pmatrix} 0 & -1 & 4 & 1 \\ 1 & 1 & 2 & -1 \\ -3 & 0 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}$.

   (a) Find the rank and the nullity of the matrix B.
   (b) Find a basis for the row space of B, then extend this basis to a basis for $\mathbb{R}^4$.

6. Show that the functions $f_1(x) = x$, $f_2(x) = xe^x$, and $f_3(x) = e^{-x}$ are linearly independent in the vector space $C^\infty(\mathbb{R})$.

7. Let V and W be subspaces of the vector space $\mathbb{R}^n$ such that $V \cup W$ is also a subspace of $\mathbb{R}^n$. Show that $V \subset W$ or $W \subset V$.