MATH 304–505 Spring 2009

Sample problems for Test II

1. Let L be the linear operator on P_3 defined by

$$L(p(x)) = xp'(x) + p''(x)$$

   (a) Find the matrix A representing L with respect to $[1, x, x^2]$.

   (b) Find the matrix B representing L with respect to $[1, x, 1 + x^2]$.

   (c) Find the matrix S such that $B = S^{-1}AS$

2. Find the distance from the point (2,1,-2) to the plane $6(x-1) + 2(y-3) + 3(z+4) = 0$.

3. Let V be a subspace spanned by vectors $\mathbf{x}_1 = (1, 1, 1, 1)$ and $\mathbf{x}_2 = (1, 0, 3, 0)$.

   (a) Find an orthonormal basis for V.

   (b) Find an orthonormal basis for $V^{\perp}$.

4. Compute $\|x\|_1$, $\|x\|_2$, and $\|x\|_\infty$ for the vector $\mathbf{x} = (-1, 3, -4)$.

5. Find the linear polynomial which is the best least squares fit to the following data

x	-2	-1	0	1	2
$f(x)$	-3	-2	1	2	5

6. Let Π be the plane spanned by the vectors $\mathbf{x}_1 = (1, 1, 0)$ and $\mathbf{x}_2 = (0, 1, 1)$. Find the orthogonal projection of the vector $\mathbf{y} = (-2, 1, 4)$ onto Π.