Math 151 Review for Test 3. Answers. Fall 2011

1. (a.) 4

   (b.) $\log_2 6$

2. $f'(x) = \dfrac{2x}{\sin^{-1}(x^2)\sqrt{1-x^4}}$

3. (a.) $f'(x) = \dfrac{\sqrt[3]{3x-1}\,(x-2)^3}{2\sqrt{x+1}}\left(\dfrac{1}{3x-1}+\dfrac{3}{x-2}-\dfrac{1}{2(x+1)}\right)$

   (b.) $f(x) = (x+x^2)^{\tan x}\left(\sec^2 x \ln(x+x^2) + \tan x \dfrac{1+2x}{x+x^2}\right)$

4. (a.) $y(t) = 4000 \cdot 3^{2t}$

   (b.) $4000 \cdot 3^{2/3}$

   (c.) $t = \dfrac{1}{2}\log_3 5$

5. $\dfrac{3\pi}{4}$

6. (a.) If $y = \sin^{-1} x$, then $x = \sin y$.

$$\frac{d}{dx}x = \frac{d}{dx}\sin y$$

$$1 = \cos y \frac{dy}{dx}$$

Then

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

If $\sin y = x$, then $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$. Therefore,

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

(b.) If $y = \cos^{-1} x$, then $x = \cos y$.

$$\frac{d}{dx}x = \frac{d}{dx}\cos y$$

$$1 = -\sin y \frac{dy}{dx}$$

Then

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

If $\cos y = x$, then $\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$. Therefore,

$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

(c.) If $y = \tan^{-1} x$, then $x = \tan y$.

$$\frac{d}{dx}x = \frac{d}{dx}\tan y$$

$$1 = \sec^2 y \frac{dy}{dx}$$

Then

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

If $\tan y = x$, then $\sec^2 y = \tan^2 y + 1 = x^2 + 1$. Therefore,

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

7. $f'(x) = \dfrac{4x}{\sqrt{1 + (\tan^{-1}(2x^2+3))^2}\ (1 + (2x^2+3)^2)}$

8. (a.) $\dfrac{1}{2}$

   (b.) 0

   (c.) 1

9. (a.) f is increasing on $(b, e) \cup (0, i) \cup (l, m)$. f is decreasing on $(a, b) \cup (e, 0) \cup (i, l)$.

   (b.) f has local maxima at $x = e, i$. f has local minima at $x = b, 0, l$.

   (c.) f is concave upward on $(a, c) \cup (f, h) \cup (k, m)$. f is concave downward on $(c, f) \cup (h, k)$.

   (d.) f has inflection points at $x = c, f, h, k$.

10. The absolute maximum value is $\dfrac{4}{27}$; the absolute minimum value is -4.

11. (a.) No vertical asymptotes. Horizontal asymptote $y = 0$ as $x \to -\infty$.

    (b.) f is increasing on $(-\infty, -2) \cup (0, \infty)$. f is decreasing on $(-2, 0)$.

    (c.) f has a local max at $x = -2$. No absolute max. f has the absolute min at $x = 0$.

    (d.) f is CU on $(-\infty, -2 - \sqrt{2}) \cup (-2 + \sqrt{2}, \infty)$. f is CD on $(-2 - \sqrt{2}, -2 + \sqrt{2})$.

    (e.) f has inflection points at $x = -2 - \sqrt{2}, -2 + \sqrt{2}$.

12. $r = h = \sqrt[3]{\dfrac{V}{\pi}}$.

13. (a.) $F(x) = \dfrac{3}{7}x^{7/3} + x + C$

    (b.) $F(x) = -\cos x + 2\tan^{-1} x + 3\sin^{-1} x + C$

14. $s(t) = \dfrac{t^4}{12} - \dfrac{t^3}{6} + t$

15. $\vec{r}(t) = < \dfrac{t^3}{3} + t + 1, \dfrac{3t^2}{2} - t + 2 >$

16. 40