Chapter 3. **Derivatives**
Section 3.1 **Derivatives**

Definition. The **derivative of a function** f **at a number** a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

if the limit exist.

Example 1. Find $f'(a)$ if $f(x) = \sqrt{2x-3}$, $a > 3/2$.

Geometric interpretation of the derivative. $f'(a)$ is the slope of the tangent line to $y = f(x)$ at the point $(a, f(a))$.

Example 2. Find an equation of the tangent line to $f(x) = \sqrt{2x-3}$ at the point (2,1).

Other interpretations of the derivative.

- $f'(a)$ is the instanteneous rate of change of $y = f(x)$ with respect to x when $x = a$.

- if $s = f(t)$ is the position function of a particle that moves along a straight line, then $f'(a)$ is the velocity of the particle at time $t = a$

A function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the **derivative of** f.

Definition. A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) if it is differentiable at every number in the interval.

Example 3. Where is the function $f(x) = |x - 2|$ differentiable?

Theorem. If f is differentiable at a, then f is continuous at a

When is the function not differentiable at $x = a$?

- f has a "corner" or "kink" at a

- f is discontinuous at a

- the curve $y = f(x)$ has a vertical tangent line at $x = a$