

Section 3.6 Implicit differentiation.

Some functions are defined implicitly by a relation between x and y , where x is the independent variable and y depends on x . In order to find the derivative of y with respect to x , we can use the method of **implicit differentiation**. This consists of differentiating both sides of the relation with respect to x and then solving the resulting equation for y' .

Example 1. Find dy/dx by implicit differentiation.

1. $x^2 - xy + y^3 = 8$

2. $\frac{y}{x-y} = x^2 + 1$

3. $\sqrt{x+y} + \sqrt{xy} = 6$

4. $x \sin y + \cos 2y = \cos y$

Example 2. Let y be the independent variable and x be the dependent variable. Use implicit differentiation to find dx/dy if

$$(x^2 + y^2)^2 = 4x^2y$$

Definition. Two curves are called **orthogonal** if at each point of intersection their tangent lines are perpendicular.

Example 3. Show that the curves $x^2 - y^2 = 5$ and $4x^2 + 9y^2 = 72$ are orthogonal.