

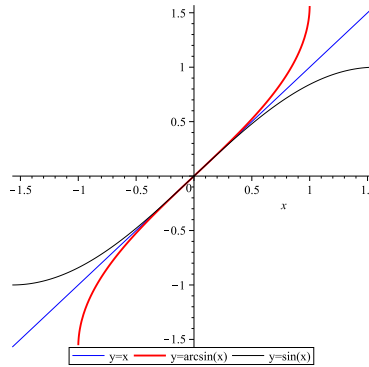
## Section 4.6 Inverse trigonometric functions

### Inverse sine function

$$\arcsin x = \sin^{-1} x = y \Leftrightarrow \sin y = x$$

DOMAIN  $-1 \leq x \leq 1$

RANGE  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



### CANCELLATION EQUATIONS

$$\sin^{-1}(\sin x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\sin(\sin^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

**Example 1.** Find

1.  $\sin^{-1}(0.5)$

2.  $\sin^{-1}(\sin 1)$

3.  $\sin(2 \sin^{-1} \frac{3}{5})$

4.  $\arcsin(\sin \frac{5\pi}{4})$

**Example 2.** Simplify  $\tan(\sin^{-1} x)$

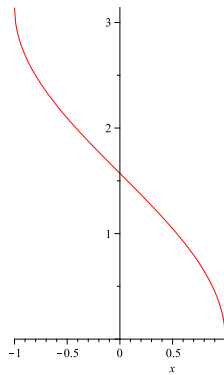
$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

**Inverse cosine function**

$$\arccos x = \cos^{-1} x = y \Leftrightarrow \cos y = x$$

DOMAIN  $-1 \leq x \leq 1$

RANGE  $0 \leq y \leq \pi$



**CANCELLATION EQUATIONS**

$$\cos^{-1}(\cos x) = x \quad \text{for } 0 \leq x \leq \pi$$

$$\cos(\cos^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

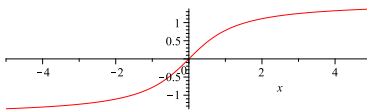
$$(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$$

**Inverse tangent function**

$$\arctan x = \tan^{-1} x = y \Leftrightarrow \tan y = x$$

DOMAIN  $-\infty \leq x \leq \infty$

RANGE  $-\frac{\pi}{2} < y < \frac{\pi}{2}$



### CANCELLATION EQUATIONS

$$\tan^{-1}(\tan x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\tan(\tan^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty$$

$$\lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow \infty} \tan^{-1} x = \frac{\pi}{2}$$

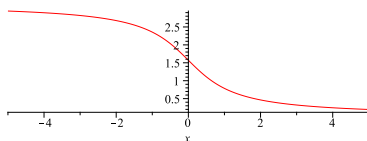
$$(\tan^{-1} x)' = \frac{1}{1+x^2}$$

### Inverse cotangent function

$$\operatorname{arccot} x = \cot^{-1} x = y \Leftrightarrow \cot y = x$$

DOMAIN  $-\infty \leq x \leq \infty$

RANGE  $0 < y < \pi$



### CANCELLATION EQUATIONS

$$\cot^{-1}(\cot x) = x \quad \text{for } 0 \leq x \leq \pi$$

$$\cot(\cot^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty$$

$$\lim_{x \rightarrow -\infty} \cot^{-1} x = 0$$

$$\lim_{x \rightarrow \infty} \cot^{-1} x = \pi$$

$$(\cot^{-1} x)' = -\frac{1}{1+x^2}$$

### Other inverse trigonometric functions

$$\csc^{-1} x = y \Leftrightarrow \csc y = x$$

DOMAIN  $|x| \geq 1$

RANGE  $y \in (0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$

$$(\csc^{-1} x)' = -\frac{1}{x\sqrt{x^2-1}}$$

$$\sec^{-1} x = y \Leftrightarrow \sec y = x$$

DOMAIN  $|x| \geq 1$

RANGE  $y \in [0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$

$$(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$$

**Example 3.** Differentiate each function:

1.  $f(x) = \sin^{-1}(2x - 1)$

2.  $g(x) = x \cos^{-1} x - \sqrt{1 - x^2}$

3.  $h(x) = \sin^{-1}(\tan^{-1} x)$

4.  $u(t) = 2^{\csc^{-1} t}$