Section 5.3 **Derivatives and the shapes of curves.**

The mean value theorem If f is a differentiable function on the interval $[a, b]$, then there exist a number c, $a < c < b$, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \text{ or } f(b) - f(a) = f'(c)(b - a)$$

Increasing/decreasing test

1. If $f'(x) > 0$ on an interval, then f is increasing on that interval
2. If $f'(x) < 0$ on an interval, then f is decreasing on that interval

The first derivative test Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local max at c.
2. If f' changes from negative to positive at c, then f has a local min at c.
3. If f' does not change sign c, then f has a no local max or min at c.

Definition. A function is called **concave upward** (CU) on an interval I if f' is an increasing function on I. It is called **concave downward** (CD) on I if f' is an decreasing on I.

A point where a curve changes its direction of concavity is called an **inflection point**.

Concavity test

1. If $f''(x) > 0$ on an interval, then f is CU on this interval.
2. If $f''(x) < 0$ on an interval, then f is CD on this interval.

The second derivative test Suppose f'' is continuous near c.

1. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local min at c.
2. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local max at c.

Example 1. Sketch the graph of the function

(a.) $f(x) = \mathrm{e}^{-\frac{1}{x+1}}$

(b.) $f(x) = \frac{x}{(x-1)^2}$