- HW over 3.11, 4.1 is due 10/26, 11:55 PM
- HW over 4.2, 4.3 is due 11/2, 11:55 PM
- Test 2 7:30-9:30 PM, ARCC 105, 10/27
  over 3.2, 3.4-3.11, 4.1
  Bring one long green scantron (822E)

1. If $f(x) = \dfrac{x^2}{h(x)}$, find $f'(2)$ if it is known that $h(2) = -4$ and $h'(2) = 3$.

(a) $-\dfrac{28}{9}$

(b) $\dfrac{1}{4}$

(c) $-\dfrac{7}{2}$

(d) $-\dfrac{7}{4}$

(e) $\dfrac{7}{4}$

$$f'(x) = \frac{2x\,h(x) - x^2 h'(x)}{[h(x)]^2}$$

$$f'(2) = \frac{2(2)\,h(2) - 2^2 h'(2)}{[h(2)]^2} = \frac{4(-4) - 4(3)}{(-4)^2}$$

$$= \frac{-28}{16} = -\frac{7}{4}$$

2. Suppose an object is moving according to the position function $s(t) = \cos t + \dfrac{t^2}{4}$, where t is measured in minutes and $s(t)$ in feet. At what time(s) is the acceleration equal to zero for $0 \le t \le 2\pi$?

(a) $t = \dfrac{\pi}{3}$ and $t = \dfrac{2\pi}{3}$

(b) $t = \dfrac{\pi}{6}$ and $t = \dfrac{11\pi}{6}$

(c) $t = \dfrac{\pi}{6}$ and $t = \dfrac{5\pi}{6}$

(d) $t = \dfrac{\pi}{3}$ and $t = \dfrac{4\pi}{3}$

(e) $t = \dfrac{\pi}{3}$ and $t = \dfrac{5\pi}{3}$

$$v(t) = -\sin t + \frac{2t}{4} = -\sin t + \frac{t}{2}$$

$$a(t) = s''(t) = -\cos t + \frac{1}{2} = 0$$

$$\cos t = \frac{1}{2}$$

$$t = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$$

$$\frac{\pi}{3},\ \frac{5\pi}{3}$$

3. Find the slope of the tangent line to the curve $\frac{d}{dx}\left(x^3 - 3xy + y^3\right) = \frac{d}{dx}3$ at the point $(2, 1)$.

(a) 3

(b) 2

(c) -3

(d) -1

(e) -2

$$\frac{3x^2 - 3y - 3xy' + 3y^2y'}{3} = \frac{0}{3}$$

$$x^2 - y - xy' + y^2y' = 0$$

$$y'(y^2 - x) = y - x^2$$

$$y' = \frac{y - x^2}{y^2 - x}$$

$$y'(2,1) = \frac{1 - 2^2}{1^2 - 2}$$

$$= \frac{-3}{-1} = 3$$

4. Find $f^{(99)}(x)$, that is the ninetyninth derivative of $f(x)$, for $f(x) = \frac{1}{x}$. $= x^{-1}$

(a) $-\frac{99!}{x^{99}}$

(b) $\frac{100!}{x^{99}}$

(c) $-\frac{99!}{x^{100}}$

(d) $-\frac{100!}{x^{99}}$

(e) $\frac{99!}{x^{100}}$

$f'(x) = (-1)x^{-2}$

$f''(x) = (-1)(-2)x^{-3}$

$= (-1)^2(1)(2)x^{-3}$

$f'''(x) = (-1)^2(1)(2)(-3)x^{-4}$

$= (-1)^3 \cdot 3! \cdot x^{-4}$

$f^{(n)}(x) = (-1)^n \cdot n!\ x^{-n-1}$

$f^{(99)}(x) = \frac{(-1)^{99}\ 99!}{x^{99+1}} = -\frac{99!}{x^{100}}$

5. At what point on the curve $f(x) = 36\sqrt{x}$ is the tangent line parallel to the line $9x - y + 2 = 0$?

(a) ~~(16, 144)~~

(b) $\left(\frac{1}{4}, 18\right)$

(c) $(4, 38)$

(d) $\left(\frac{1}{16}, 9\right)$

(e) $(4, 72)$

$y = 9x + 2$

$m = 9$

$f'(x) = 36\left(\frac{1}{2}\right)x^{-1/2}$

$= \frac{18}{\sqrt{x}} = 9$

$\frac{2}{\sqrt{x}} = 1 \Rightarrow \sqrt{x} = 2$

$x = 4$, $y = f(4) = 36\sqrt{4} = 72$

$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

6. Find the quadratic approximation for $f(x) = xe^{3x}$ at $x = 0$.

(a) $x + 6x^2$

(b) $1 + x + x^2$

(c) $x + 3x^2$

(d) $1 + x + \frac{c^2}{2}$

(e) $x + 2x^2$

$f(x) = xe^{3x}$ | $f(0) = 0$

$f'(x) = e^{3x} + x\left(e^{3x}\right)'$ | $f'(0) = 1$

$= e^{3x} + xe^{3x}(3x)'$

$= e^{3x} + 3xe^{3x}$

$f''(x) = 3e^{3x} + 3e^{3x} + 3x\left(e^{3x}\right)'$ | $f''(0) = 6$

$= 6e^{3x} + 3x\left(3e^{3x}\right)$

$= 6e^{3x} + 9xe^{3x}$

$0 + (1)(x-0) + \frac{6}{3}(x-0)^2$

$= x + 2x^2$

7. If $H(x) = xf(p(x))$, find $H'(3)$ if it is known that $p(3) = 7$, $f'(7) = 2$, $f(7) = 4, f'(3) = 5$ and $p'(3) = -1$.

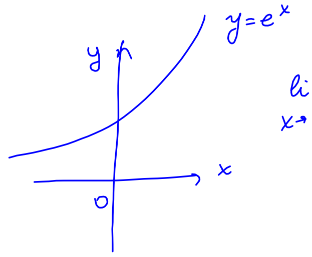
(a) -2
(b) 10
(c) -6
(d) -38
(e) 12

$$H'(x) = f(p(x)) + x[f(p(x))]'$$
$$= f(p(x)) + x f'(p(x)) \cdot p'(x)$$
$$H'(3) = f(p(3)) + 3 f'(p(3))\, p'(3)$$
$$= f(7) + 3 f'(7)(-1)$$
$$= 4 + 3(2)(-1) = 4 - 6 = -2$$

8. Find a tangent vector for $\mathbf{r}(t) = \langle t\cos(2t), e^{5t} \rangle$ at $t = \pi$.

(a) $\langle -1, 5e^{5\pi} \rangle$
(b) $\langle 1, e^{5\pi} \rangle$
(c) $\langle -2\pi, 5e^{5\pi} \rangle$
(d) $\langle 1, 5e^{5\pi} \rangle$
(e) $\langle \pi, e^{5\pi} \rangle$

$$\vec{r}'(t) = \langle \cos 2t + t(\cos 2t)', 5e^{5t} \rangle$$
$$= \langle \cos 2t + t(-\sin 2t)(2), 5e^{5t} \rangle$$
$$= \langle \cos 2t - 2t \sin 2t, 5e^{5t} \rangle$$
$$r'(\pi) = \langle \cos 2\pi - 2\pi \sin 2\pi, 5e^{5\pi} \rangle$$
$$= \langle 1, 5e^{5\pi} \rangle$$



$$\lim_{x \to -\infty} e^x = 0$$

9. $\displaystyle\lim_{x \to 0^-} \frac{3}{1 + e^{1/x}} =$

(a) $-\infty$
(b) ∞
(c) 0
(d) 1
(e) 3

$$t = \frac{1}{x}$$
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$
$$= \lim_{t \to -\infty} \frac{3}{1 + e^t} = 3$$

10. $\lim\limits_{x\to 2} \dfrac{\sin(x-2)}{x^2+2x-8} = \lim\limits_{x\to 2} \dfrac{\sin(x-2)}{(x-2)(x+4)} = \lim\limits_{x\to 2} \overbrace{\dfrac{\sin(x-2)}{x-2}}^{1} \cdot \dfrac{1}{x+4} = \dfrac{1}{2+4} = \dfrac{1}{6}$

(a) 0

(b) $\frac{1}{9}$

(c) 1

(d) $\frac{1}{6}$

(e) The limit does not exist.

11. Find $f''(1)$ if $f(x) = (3x-1)^5$

(a) 960

(b) 1440

(c) 480

(d) 240

(e) 160

$f'(x) = 5(3x-1)^4 \overbrace{(3x-1)'}^{3}$
$= 15(3x-1)^4$

$f''(x) = 15(4)(3x-1)^3 \overbrace{(3x-1)'}^{3}$
$= 180(3x-1)^3$

$f''(1) = 180(3-1)^3$
$= 180(8) = 1440$

12. What is the slope of the parametric curve $x = t^2 + t + 2$, $y = 4 - 7t$ at the point $(4, -3)$?

(a) -1

(b) $\frac{7}{3}$

(c) $-\frac{7}{3}$

(d) $\frac{3}{7}$

(e) $-\frac{3}{7}$

Find t such that $\begin{cases} t^2+t+2=4 \\ 4-7t=-3 \end{cases}$ $\underline{t=1}$

$\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)} = \dfrac{-7}{2t+1}$

$\left.\dfrac{dy}{dx}\right|_{t=1} = -\dfrac{7}{2+1} = -\dfrac{7}{3}$

13. Find all point(s) on the curve defined by the parametric equations $x = t^3 - 3t - 1$ and $y = t^3 - 12t + 3$ where the tangent line is vertical.

vertical tangent $\Rightarrow x'(t) = 0$

$x'(t) = 3t^2 - 3 = 0$

$t^2 - 1 = 0$

$t = \pm 1$

$x(1) = 1-3-1 = -3$ $\quad$ $x(-1) = -1+3-1 = 1$

$y(1) = 1-12+3 = -8$ $\quad$ $y(-1) = -1+12+3 = 14$

$(-3,-8)$ $\quad$ $(1,14)$

(a) $(-3, -8)$ and $(1, 14)$

(b) $(1, -1)$

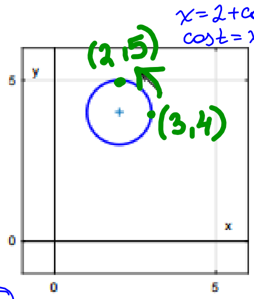
(c) $(-3, -8)$

(d) $(1, -13)$ and $(-3, 19)$

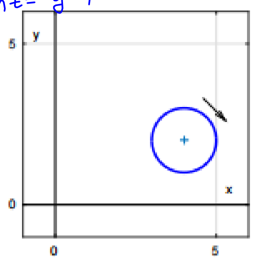
(e) $(1, -13)$

14. Sketch the graph of $\langle 2 + \cos t, \sin(t) + 4\rangle$ and indicate the direction of the curve as t increases. Section 1.3

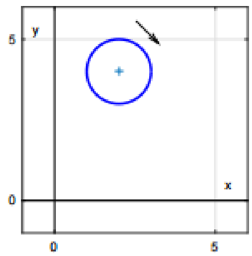
$x = 2 + \cos t$, $\cos t = x - 2$, $\quad$ $y = \sin t + 4$, $\sin t = y - 4$



(a)



(b)



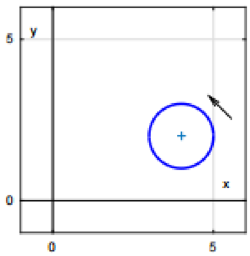
(c)

$\sin^2 t + \cos^2 t = 1$

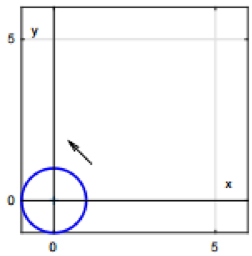
$(x-2)^2 + (y-4)^2 = 1$

circle

center @ $(2,4)$



(d)



(e)

$\begin{cases} x = 2 + \cos t \\ y = \sin t + 4 \end{cases}$

$t = 0$ $\quad$ $x(0) = 2+1 = 3$, $y(0) = 4+0 = 4$ $\quad$ $(3,4)$

$t = \pi/2$ $\quad$ $x = 2+0 = 2$, $y = 1+4 = 5$ $\quad$ $(2,5)$

15. An object is moving along a straight path. The position of the object at time t is given by $s(t) = 2t^3 - 9t^2 + 12t + 1$, where t is measured in seconds and $s(t)$ is measured in feet. Find the total distance traveled in the first 2 seconds.

$v(t) = s'(t) = 6t^2 - 18t + 12$

$= 6(t^2 - 3t + 2) > 0$

$6(t-2)(t-1) > 0$

(a) 4 feet

(b) 3 feet

(c) 8 feet

(d) 5 feet

(e) 6 feet

$+ \quad 1 \quad - \quad 2 \quad +$

$t=0$ $\quad$ $t=1$ $\quad$ $t=2$

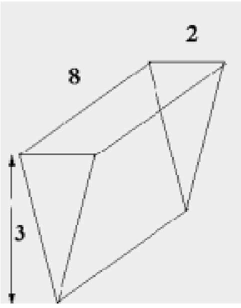
$|s(1) - s(0)| + |s(2) - s(1)|$

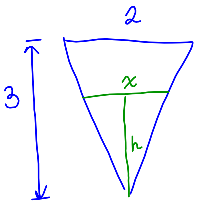
$|2-9+12+1-1| + |2(8) - 9(4) + 12(2) + 1 - (2-9+12+1)|$

$5 + |(16 - 36 + 24 + \cancel{1} - 2 + 9 - 12 - \cancel{1})|$

$5 + 1 = 6$

16. (8 pts) A trough 8 feet long, 3 feet high and 2 feet across the top is being filled with water at a rate $\frac{1}{10}$ cubic feet per minute. How fast is the water level rising when the height of the water is 1 foot?





similar Δ:

$$\frac{2}{x} = \frac{3}{h}$$

$$2h = 3x$$

$$x = \frac{2}{3}h$$

$$\frac{dV}{dt} = \frac{1}{10}$$

$$V = \frac{1}{2} h x \cdot 8$$

$$= 4hx$$

$$\frac{dh}{dt} = ?$$

$$V = 4h\left(\frac{2}{3}h\right)$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{8h^2}{3}\right)$$

$$\frac{dV}{dt} = \frac{8}{3} 2h \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{3}{16h} \frac{dV}{dt}$$

$$= \frac{3}{16} \frac{1}{10} = \boxed{\frac{3}{160} \text{ ft/min}}$$

17. (6 pts) Use an appropriate linear (tangent) approximation to estimate $\sqrt[3]{8.01}$. $= \sqrt[3]{8+.01}$

$$f(a+\Delta x) \approx f(a) + f'(a)\Delta x$$

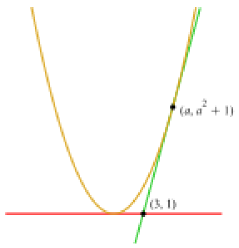
$$f(x) = \sqrt[3]{x}, \quad a = 8, \quad \Delta x = .01$$

$$f(8) = 2$$

$$f'(x) = \frac{1}{3}x^{-2/3}, \quad f'(8) = \frac{1}{3 \cdot 8^{2/3}} = \frac{1}{12}$$

$$\sqrt[3]{8.01} \approx 2 + \frac{.01}{12} = \frac{24.01}{12} = \boxed{\frac{2401}{1200}}$$

18. (7 pts) The horizontal line through the point $(3, 1)$ is tangent to the parabola $y = x^2 + 1$. We see from the figure below that there is a second line tangent to the parabola at $(a, a^2 + 1)$ that also passes through the point $(3, 1)$. Find the value of a.



Write down an equation of a tangent line to $y = x^2+1$ @ (a, a^2+1)

$$y = f'(a)(x-a) + (a^2+1)$$

$$f(x) = x^2+1, \quad f'(x) = 2a, \quad f'(a) = 2a$$

$y = 2a(x-a) + (a^2+1)$ passes through $(3,1)$

$$1 = 2a(3-a) + a^2 + 1$$

$$\cancel{1} = 6a - 2a^2 + a^2 + \cancel{1}$$

$$6a - a^2 = 0$$

$$a(6-a) = 0$$

$$a = 0, \quad \boxed{a = 6}$$

$a=0$

$y = 1$ - horizontal tangent

19. Find $f'(x)$. Do not simplify.

(a) (6 pts) $f(x) = x\sin^7(\cos(6x))$

$$f'(x) = \sin^7(\cos 6x) + x\left[\sin^7(\cos 6x)\right]'$$
$$= \sin^7(\cos 6x) + x\,(7)\sin^6(\cos 6x)\left[\sin(\cos 6x)\right]'$$
$$= \sin^7(\cos 6x) + 7x\sin^6(\cos 6x)\cos(\cos 6x)(\cos 6x)'$$
$$= \sin^7(\cos 6x) + 7x\sin^6(\cos 6x)\cos(\cos 6x)(-\sin 6x)(6x)'$$
$$= \boxed{\sin^7(\cos 6x) + 7x\sin^6(\cos 6x)\cos(\cos 6x)(-\sin 6x)(6)}$$

(b) (6 pts) $f(x) = \dfrac{(x-1)^2}{e^{x^2+2x}}$

$$f'(x) = \frac{2(x-1)e^{x^2+2x} - (x-1)^2\left(e^{x^2+2x}\right)'}{\left(e^{x^2+2x}\right)^2} = \frac{2(x-1)e^{x^2+2x} - (x-1)^2 e^{x^2+2x}(x^2+2x)'}{e^{2x^2+4x}}$$

$$= \frac{2(x-1)\cancel{e^{x^2+2x}} - (x-1)^2\cancel{e^{x^2+2x}}(2x+2)}{\left(e^{x^2+2x}\right)^{\cancel{2}}}$$

$$= \boxed{\frac{2(x-1) - (x-1)^2(2x+2)}{e^{x^2+2x}}}$$

20. (7 pts) Find $\frac{dy}{dx}$ if $\frac{d}{dx}\left(\tan(xy^2) + \sin y\right) = \frac{d}{dx}\left(6x^2 + 8y + 2\right)$

$$\sec^2(xy^2)\,\frac{d}{dx}(xy^2) + \cos y \cdot y' = 12x + 8y'$$

$$\sec^2(xy^2)\left[y^2 + 2xy\,y'\right] + \cos y\, y' - 8y' = 12x$$

$$y'\left[2xy\sec^2(xy^2) + \cos y - 8\right] = 12x - y^2\sec^2(xy^2)$$

$$\boxed{y' = \frac{12x - y^2\sec^2(xy^2)}{2xy\sec^2(xy^2) + \cos y - 8}}$$