Appendix D. **Trigonometry.**

Angles can be measured in degrees or radians. The angle given by a complete revolution contains $360^\circ$, or $2\pi$ radians.

$$360^\circ = 2\pi \text{ rad}$$

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

**Example 1.**

(a.) Convert $9^\circ$ to radians.

(b.) Convert $\frac{5\pi}{12}$ to degrees.

The **standard position** of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive $x$-axis. A **positive** angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side.

**Negative** angles are obtained by clockwise rotation.
The trigonometric functions.

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}
\]
\[
\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}
\]
\[
\sec \theta = \frac{\text{hyp}}{\text{adj}} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}}
\]

Special triangles:

\[
\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \sin \frac{\pi}{6} = \frac{1}{2} \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}
\]
\[
\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \cos \frac{\pi}{3} = \frac{1}{2}
\]
\[
\tan \frac{\pi}{4} = 1 \quad \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \quad \tan \frac{\pi}{3} = \sqrt{3}
\]
Signs of the trigonometric functions:

\[
\begin{array}{c c}
\text{sin } \theta & > 0 \\
\text{all ratios} & > 0 \\
\text{tan } \theta & > 0 \\
\cos \theta & > 0
\end{array}
\]

Example 2.
(a.) If \( \sin \theta = \frac{3}{5} \) \((0 < \theta < \pi/2)\), find the remaining trigonometric ratios.

(b.) If \( \cos \theta = -\frac{1}{3} \) \((\pi < \theta < 3\pi/2)\), find the remaining trigonometric ratios.
Trigonometric Identities:

\[
\begin{align*}
\csc \theta & = \frac{1}{\sin \theta} & \sec \theta & = \frac{1}{\cos \theta} & \cot \theta & = \frac{1}{\tan \theta} & \tan \theta & = \frac{\sin \theta}{\cos \theta} \\
\sin^2 \theta + \cos^2 \theta & = 1 & \tan^2 \theta + 1 & = \sec^2 \theta \\
\cot^2 \theta + 1 & = \csc^2 \theta \\
\sin(-\theta) & = -\sin \theta & \cos(-\theta) & = \cos \theta \\
\sin(\theta + 2\pi) & = \sin \theta & \cos(\theta + 2\pi) & = \cos \theta \\
\tan(\theta + \pi) & = \tan \theta & \cot(\theta + \pi) & = \cot \theta \\
\sin(x + y) & = \sin x \cos y + \cos x \sin y \\
\sin(x - y) & = \sin x \cos y - \cos x \sin y \\
\cos(x + y) & = \cos x \cos y - \sin x \sin y \\
\cos(x - y) & = \cos x \cos y + \sin x \sin y \\
\tan(x + y) & = \frac{\tan x + \tan y}{1 - \tan x \tan y} \\
\tan(x - y) & = \frac{\tan x - \tan y}{1 + \tan x \tan y} \\
\sin 2x & = 2 \sin x \cos x \\
\cos 2x & = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1 \\
\cos^2 x & = \frac{1 + \cos 2x}{2} \\
\sin^2 x & = \frac{1 - \cos 2x}{2} \\
\sin x \cos y & = \frac{1}{2}[\sin(x + y) + \sin(x - y)] \\
\cos x \cos y & = \frac{1}{2}[\cos(x + y) + \cos(x - y)] \\
\sin x \sin y & = \frac{1}{2}[\cos(x - y) - \cos(x + y)]
\end{align*}
\]

Example 4. If \( \frac{1}{3} \frac{5}{4} \), where \( x \) and \( y \) lie between 0 and \( \pi/2 \), evaluate the expression
(a.) \( \cos(x - y) \)
Example 5. Find all values of $x$ in the interval $[0, 2\pi]$ that satisfy the equation:

(a.) $2\cos x - 1 = 0$

(b.) $2\cos x + \sin 2x = 0$
Graphs of the trigonometric functions.

\[ y = \sin x \]

\[ y = \cos x \]

\[ y = \tan x \quad y = \cot x \]
Example 6. Sketch the graph of $f(x) = 1 - \sin x$