Appendix D. **Trigonometry.**

Angles can be measured in degrees or radians. The angle given by a complete revolution contains 360^o, or 2π radians.

$$360^o = 2\pi \text{ rad}$$

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^o$$

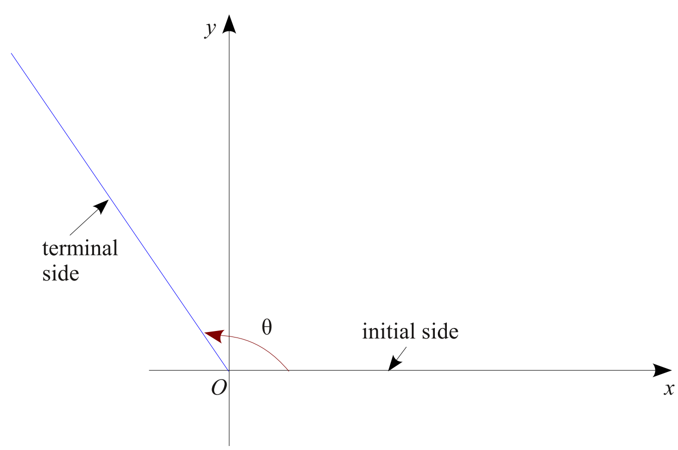
$$1^o = \frac{\pi}{180} \text{ rad}$$

Example 1.

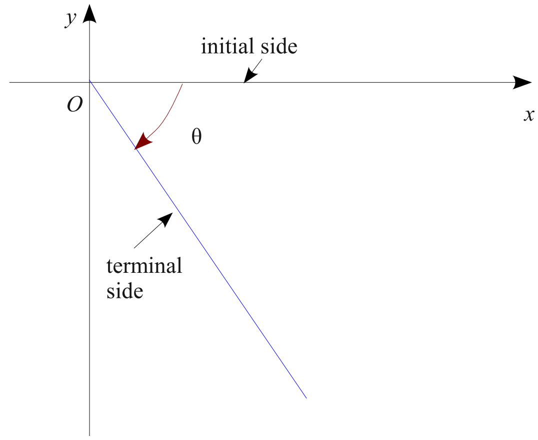
(a.) Convert 9^o to radians.

(b.) Convert $\frac{5\pi}{12}$ to degrees.

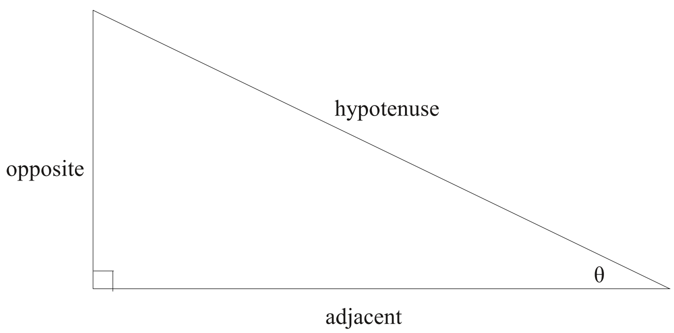
The **standard position** of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis. A **positive** angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side.



Negative angles are obtained by clockwise rotation

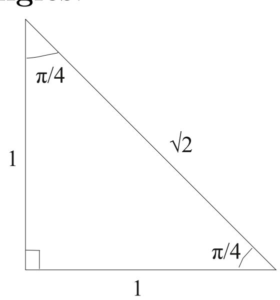


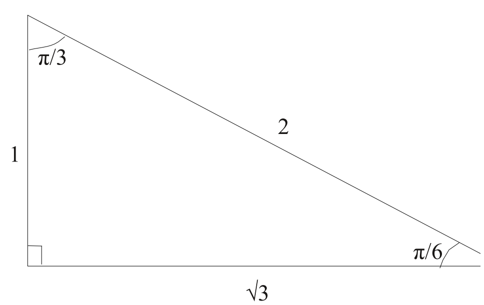
The trigonometric functions.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

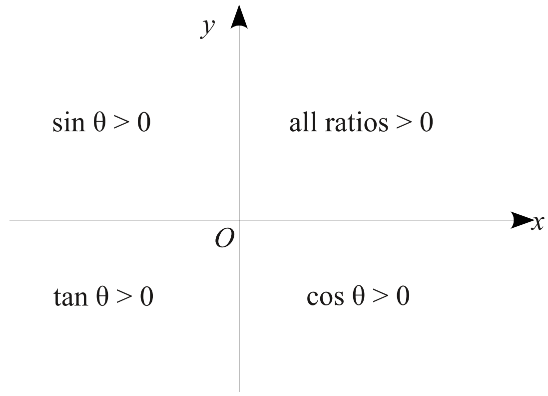
Special triangles:





$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$
$$\tan\frac{\pi}{4} = 1 \qquad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \qquad \sin\frac{\pi}{3} = \sqrt{3}$$

Signs of the trigonometric functions:



Example 2.

(a.) If $\sin\theta = \frac{3}{5}$ $(0 < \theta < \pi/2)$, find the remaining trigonometric rations.

(b.) If $\cos\theta = -\frac{1}{3}$ $(\pi < \theta < 3\pi/2)$, find the remaining trigonometric rations.

Trigonometric Identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\sin(\theta + 2\pi) = \sin\theta \qquad \cos(\theta + 2\pi) = \cos\theta$$

$$\tan(\theta + \pi) = \tan\theta \qquad \cot(\theta + \pi) = \cot\theta$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$

$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin x\cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x\cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin x\sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

Example 4. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate the expression

(a.) $\cos(x-y)$

(b.) $\sin 2y$

Example 5. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation:
(a.) $2\cos x - 1 = 0$

(b.) $2\cos x + \sin 2x = 0$

Graphs of the trigonometric functions.

$y = \sin x$

$y = \cos x$

$y = \tan x$

$y = \cot x$

$y = \csc x$

$y = \sec x$

Example 6. Sketch the graph of $f(x) = 1 - \sin x$