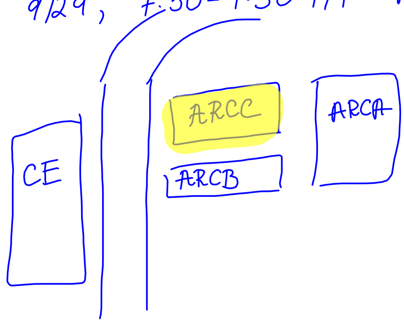
9/29, 7:30-9:30 PM ARCC 105



20 MC ← 3.5 points each
4 Work-out

no calculators!

scantron 882-E
long, green one

if u score <70 points, u can take the retest

retest Friday, Oct. 7. 25 MC questions

1. Find the average rate of change of $f(x) = \sqrt{x-1}$ from $x = 5$ to $x = 10$.

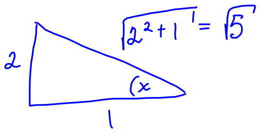
   (a) $-\frac{1}{5}$
   (b) 1
   (c) $\frac{1}{5}$
   (d) $\frac{\sqrt{5}}{5}$
   (e) $\frac{1}{4}$

$$\frac{f(10)-f(5)}{10-5} = \frac{\sqrt{10-1} - \sqrt{5-1}}{5} = \frac{\sqrt{9}-\sqrt{4}}{5} = \frac{1}{5}$$

III quadrant

2. If $\tan x = 2$ and $\underbrace{\pi < x < \frac{3\pi}{2}}$, find the value of $\cos x$.

   (a) $\frac{1}{\sqrt{5}}$
   (b) $-\frac{1}{\sqrt{10}}$
   (c) $\frac{2}{\sqrt{5}}$
   (d) $-\frac{1}{\sqrt{5}}$
   (e) $-\frac{2}{\sqrt{5}}$



$$\cos x = -\frac{1}{\sqrt{5}}$$

3. If $\mathbf{a} = \langle 1, 3\rangle$ and $\mathbf{b} = \langle 2, 2\rangle$, find $|3\mathbf{a} - 2\mathbf{b}|$

   (a) 1
   (b) $\sqrt{3}$
   (c) 4
   (d) $\sqrt{24}$
   (e) $\sqrt{26}$

$$3\vec{a} - 2\vec{b} = 3\langle 1,3\rangle - 2\langle 2,2\rangle$$
$$= \langle 3,9\rangle - \langle 4,4\rangle = \langle 3-4, 9-4\rangle = \langle -1,5\rangle$$
$$|3\vec{a}-2\vec{b}| = \sqrt{(-1)^2+5^2} = \sqrt{26}$$

$\sqrt{x^2} = -x$

4. Evaluate $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{9x^2+12x-7}}{-2x+2}$.

$$= \lim_{x\to-\infty} \frac{\sqrt{x^2\left(9+\cancel{\frac{12}{x}}^{\,0} - \cancel{\frac{7}{x^2}}^{\,0}\right)}}{x\left(-2+\cancel{\frac{2}{x}}^{\,0}\right)} = \lim_{x\to-\infty} \frac{\sqrt{9x^2}}{-2x} = \lim_{x\to-\infty}\frac{-3x}{-2x} = \frac{3}{2}$$

(a) $-\frac{3}{2}$
(b) $\frac{3}{2}$
(c) 0
(d) $\frac{9}{2}$
(e) $-\frac{9}{2}$

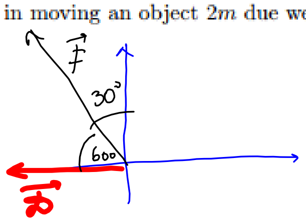
5. Find the work done by a force $\mathbf{F}$ of $25N$ acting in the direction $N30°W$ in moving an object $2m$ due west.

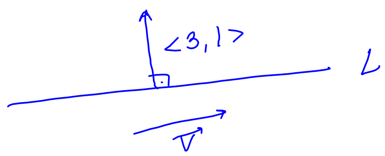
(a) $25J$
(b) $25\sqrt{3}J$
(c) $50J$
(d) $50\sqrt{3}J$
(e) None of these

$|\vec{F}| = 25$, $|\vec{D}| = 2$

$W = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos 60°$

$= (25)(2)\ \frac{1}{2} = 25$





6. Which of the following vectors is parallel to the line $y = -3x - 10$.

(a) $\mathbf{i} + 3\mathbf{j}$ <1,3>
(b) $-\mathbf{i} - 3\mathbf{j}$ <-1,-3>
(c) $-2\mathbf{i} + 6\mathbf{j}$ <-2,6>
(d) $2\mathbf{i} + 6\mathbf{j}$ <2,6>
(e) None of these

$1y + 3x = -10$, line is perpendicular to

$\langle 3, 1\rangle$

$\langle 1,3\rangle\cdot\langle 3,1\rangle \neq 0$

$\langle -1,-3\rangle\cdot\langle 3,1\rangle = -3-3 \neq 0$

$\langle -2,6\rangle\cdot\langle 3,1\rangle = 0$

$\vec{u} = \langle 3,1\rangle$, $\vec{u}^{\perp} = \langle -1,3\rangle$

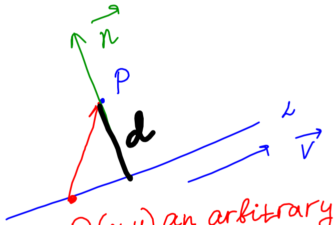
$3x - y - 4 = 0$

7. Find the distance from the point $(1, 2)$ to the line $y = 3x - 4$.

(a) $\frac{19}{\sqrt{10}}$
(b) $\frac{3}{\sqrt{10}}$
(c) $\frac{3}{\sqrt{37}}$
(d) $\frac{9}{\sqrt{10}}$
(e) $\frac{19}{\sqrt{37}}$

$y = Ax + By + C, \quad P(x_1, y_1)$

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$



$Q(x, y)$ an arbitrary point on the line

$\langle A, B \rangle = \vec{n} \perp$ line

$\vec{v} \parallel$ line

$$d = \text{comp}_{\vec{n}} \overrightarrow{QP}$$

$$d = \text{comp}_{\vec{v}\perp} \overrightarrow{QP}$$

$$d = \left| \frac{3(1) - (2) - 4}{\sqrt{3^2 + (-1)^2}} \right| = \frac{3}{\sqrt{10}}$$

8. Find $\lim_{x \to 0^-} \frac{x + 0.1}{x(x+2)}$.

$x < 0$ say $x = -0.99$

$$\frac{-0.99 + 0.1}{-0.99(-0.99 + 2)} \quad \frac{-}{(-)(+)} = +$$

(a) 0.1
(b) ∞
(c) $-\infty$
(d) 0
(e) None of these

9. Which of following intervals must contain a solution to the equation $2x^3 + 16x + 3 = 22$?

   (a) ~~$[-2, -1]$~~
   (b) ~~$[-1, 0]$~~
   (c) ~~$[0, 1]$~~
   (d) $[1, 2]$
   (e) $[2, 3]$

$2x^3+16x+3-22=0$

$\underbrace{2x^3+16x-19=0}$

$f(x)=2x^3+16x-19$

$f(-2)=2(-8)+16(-2)-19<0$

$f(-1)=2(-1)+16(-1)-19<0$

$f(0)=-19<0$

$f(1)=2+16-19<0$

$f(2)=2(8)+16(2)-19>0$

$f(3)$

10. Find the parametric equation for the line with slope $\frac{2}{3}$ and passing through the point $(-2, 3)$.

   (a) $x = -2 + 2t,\ y = 3 + 3t$
   (b) ~~$x = 2 - 2t,\ y = 3 + 3t$~~
   (c) $x = -2 + 3t,\ y = 3 - 2t$
   (d) $x = -2 - 3t,\ y = 3 + 2t$
   (e) $x = -2 + 3t,\ y = 3 + 2t$

$y=\frac{2}{3}(x+2)+3$

$3y=2(x+2)+9$

Param. eqn.

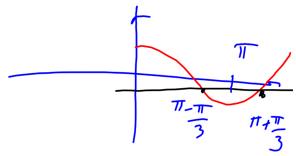
$\begin{cases} y=3+2t \\ x=-2+3t \end{cases}$

11. Given the points $A(1, -3)$ and $B(5, 1)$, find the length of a vector $\overrightarrow{AB}$.

   (a) $4\sqrt{2}$
   (b) $2\sqrt{5}$
   (c) $\sqrt{10}$
   (d) $\sqrt{26}$
   (e) 16

$\overrightarrow{AB}=\langle 5-1, 1-(-3)\rangle=\langle 4,4\rangle.$

$|\overrightarrow{AB}|=\sqrt{16+16}=\sqrt{32}=\sqrt{16\cdot 2}=4\sqrt{2}$



12. Find the **SUM** of all the solutions in $[0, 2\pi]$. If there is only one answer, give it.

$$2\cos^2 x + 3\cos x + 1 = 0$$

(a) $\frac{2\pi}{3}$
(b) $\frac{4\pi}{3}$
(c) π
(d) 2π
(e) 3π

$t = \cos x$, $-1 \le t \le 1$

$2t^2 + 3t + 1 = 0$

$t_1 = \frac{-3 + \sqrt{9-8}}{4} = -\frac{1}{2}$, $t_2 = \frac{-3-\sqrt{1}}{4} = -1$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$

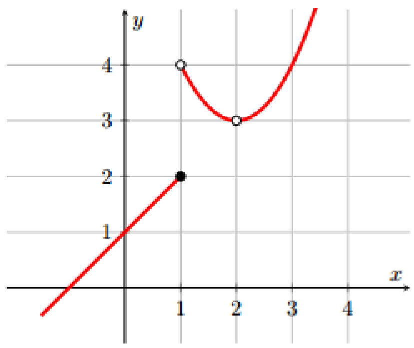
$x = \pi + \frac{\pi}{3}$ $\qquad$ $x = \pi$

$x = \pi - \frac{\pi}{3}$

$\left(\pi + \frac{\pi}{3}\right) + \left(\pi - \frac{\pi}{3}\right) + \pi = 3\pi$

---

13. Here is the graph of a function $f(x)$. Which of the following is false?



$\lim_{x \to 2^-} f(x) = 2$

$\lim_{x \to 2^+} f(x) = 4$, $2 \neq 4$

$\lim_{x \to 2} f(x)$ DNE

(a) $\lim_{x \to 1} f(x) = 2$ false
(b) $f(x)$ is continuous from the left at $x = 1$
(c) $f(1) = 2$
(d) $\lim_{x \to 2} f(x) = 3$
(e) $f(x)$ has a removable discontinuity at $x = 2$

$x+4>0 \Rightarrow x>-4$

14. Find the domain of $f(x) = \dfrac{\sqrt[3]{1-x}}{\sqrt{x+4}}$.

(a) $(-\infty, 1]$
(b) $[-4, 1]$
(c) $(-4, 1]$
(d) $[-4, 1)$
(e) $(-4, \infty)$

15. Find the vector of length 4 that is in the same direction as the vector $\langle 5, 12\rangle$.

$|\langle 5,12\rangle| = \sqrt{25+144} = \sqrt{169} = 13$

(a) $\left\langle \frac{5}{\sqrt{13}}, \frac{12}{\sqrt{13}} \right\rangle$
(b) $\left\langle \frac{20}{\sqrt{13}}, \frac{48}{\sqrt{13}} \right\rangle$
(c) $\langle \frac{5}{13}, \frac{12}{13} \rangle$
(d) $\langle \frac{20}{13}, \frac{48}{13} \rangle$
(e) $\langle \frac{10}{13}, \frac{24}{13} \rangle$

$\vec{u} = \dfrac{\langle 5,12\rangle}{13}, \quad 4\vec{u} = \dfrac{4}{13}\langle 5,12\rangle$

$= \left\langle \dfrac{20}{13}, \dfrac{48}{13} \right\rangle$

PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

if $x<0$, then $\sqrt{x^2} = -x$

16. (9 pts) Evaluate these limits. Do not use the L'Hospital method.

(a) $\displaystyle \lim_{x\to-\infty} \frac{(\sqrt{x^2+3x+1}+x)(\sqrt{x^2+3x+1}-x)}{\sqrt{x^2+3x+1}-x} = \lim_{x\to-\infty} \frac{x^2+3x+1-x^2}{\sqrt{x^2+3x+1}-x} = \lim_{x\to-\infty} \frac{3x+1}{\sqrt{x^2(1+\cancel{\frac{3}{x}}^{0}+\cancel{\frac{1}{x^2}}^{0})}-x}$

$\displaystyle = \lim_{x\to-\infty} \frac{3x+1}{\underbrace{\sqrt{x^2}}_{-x}-x} = \lim_{x\to-\infty} \frac{3x+1}{-2x} = \lim_{x\to-\infty} \frac{\cancel{x}(3+\cancel{\frac{1}{x}}^{0})}{-2\cancel{x}} = \boxed{-\frac{3}{2}}$

(b) $\displaystyle \lim_{x\to-3^-} \frac{2x^2+6x}{|x+3|} = \lim_{x\to-3^-} \frac{2x(x+3)}{|x+3|} = \lim_{x\to-3^-} \frac{2x\cancel{(x+3)}}{-\cancel{(x+3)}} = \frac{2(-3)}{-1} = \boxed{6}$

$|x+3| = \begin{cases} x+3, & \text{if } x \geq -3 \\ -(x+3), & \text{if } x < -3 \end{cases}$

(c) $\lim_{x\to 3} \frac{(x-3)(\sqrt{x^2+7}+4)}{(\sqrt{x^2+7}-4)(\sqrt{x^2+7}+4)} = \lim_{x\to 3} \frac{(x-3)(\sqrt{x^2+7}+4)}{x^2+7-16} = \lim_{x\to 3} \frac{(x-3)(\sqrt{x^2+7}+4)}{x^2-9}$

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(\sqrt{x^2+7}+4)}{\cancel{(x-3)}(x+3)} = \frac{\sqrt{3^2+7}+4}{3+3} = \frac{8}{6} = \boxed{\frac{4}{3}}$

17. (6 pts) Consider the vector function $\mathbf{r}(t) = \langle 4 + \cos t, -1 + \sin t \rangle$, $0 \le t \le 2\pi$

(a) Eliminate the parameter to find a cartesian equation.

$x(t) = 4 + \cos t$

$y(t) = -1 + \sin t$

$\cos t = x - 4$

$\sin t = y + 1$

$\sin^2 t + \cos^2 t = 1$

$\underbrace{(x-4)^2 + (y+1)^2 = 1}$

circle of radius 1 centered @ $(4, -1)$

(b) Sketch the curve on the grid below. Include the DIRECTION of the curve as <u>t increases</u>.

$x(t) = 4 + \cos t$

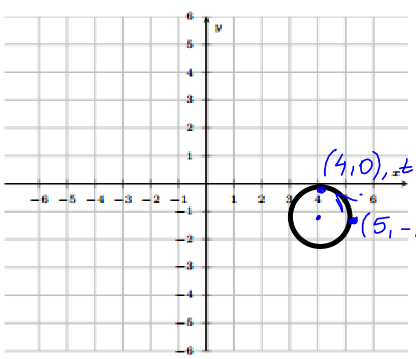
$y(t) = -1 + \sin t$

$t = 0$ $\quad x(0) = 4 + 1 = 5$

$y(0) = -1 + 0 = -1$

$t = \frac{\pi}{2}$ $\quad x\left(\frac{\pi}{2}\right) = 4 + 0 = 4$

$y\left(\frac{\pi}{2}\right) = -1 + \sin\frac{\pi}{2} = 0$



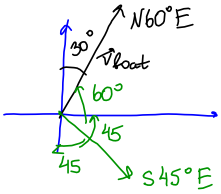
CCW

$(4, 0)$, $t = \frac{\pi}{2}$

$(5, -1)$, $t = 0$

18. (8 pts) A boat heads in the direction $N30°E$ with a speed of $40mph$. The water current is flowing $S45°E$ with a speed of $6mph$.

(a) Find the true vector velocity of the boat. (relative to the shore)



$|\vec{v}_{boat}| = 40$

$\vec{v}_{boat} = 40 < \cos 60, \sin 60 >$

$= 40 < \frac{1}{2}, \frac{\sqrt{3}}{2} > = < 20, 20\sqrt{3} >$

$|\vec{v}_{current}| = 6$

$\vec{v}_{current} = 6 < \cos 45°, -\sin 45° >$

$= 6 < \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} > = < 3\sqrt{2}, -3\sqrt{2} >$

$\vec{v} = \vec{v}_{boat} + \vec{v}_{current} = <20, 20\sqrt{3}> + <3\sqrt{2}, -3\sqrt{2}>$

$= \boxed{< 20 + 3\sqrt{2}, 20\sqrt{3} - 3\sqrt{2} >}$

(b) Find the true speed of the b~~oat. (you do not need to simplify)~~

$|\vec{v}| = \boxed{\sqrt{(20+3\sqrt{2})^2 + (20\sqrt{3} - 3\sqrt{2})^2}}$

$f(x+h) = (x+h)^2 - 8(x+h) + 9$

19. (10 pts) Find $f'(x)$ using the **limit definition of the derivative** for $f(x) = x^2 - 8x + 9$. Show all your work.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) + 9 - (x^2 - 8x + 9)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{8x} - 8h + \cancel{9} - \cancel{x^2} + \cancel{8x} - \cancel{9}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 8)}{\cancel{h}} = \boxed{2x - 8}$$

20. (7 pts) Find the value(s) of c that will make the function $g(x)$ continuous. If there are not values possible, then explain why.

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x \leq 4 \\ cx + 20 & \text{if } x > 4 \end{cases}$$

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4^+} g(x)$$

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4} (x^2 - c^2) = 4^2 - c^2 = 16 - c^2$$

$$\lim_{x \to 4^+} g(x) = \lim_{x \to 4} (cx + 20) = 4c + 20$$

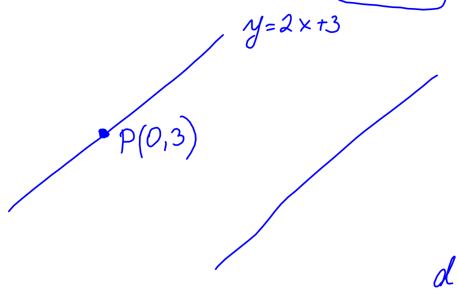
$$16 - c^2 = 4c + 20$$

$$c^2 + 4c + 20 - 16 = 0$$

$$c^2 + 4c + 4 = 0$$

$$(c+2)^2 = 0 \Rightarrow \boxed{c = -2}$$

7. Find the distance between the parallel lines $y = 2x + 3$ and $y - 2x = 9$.

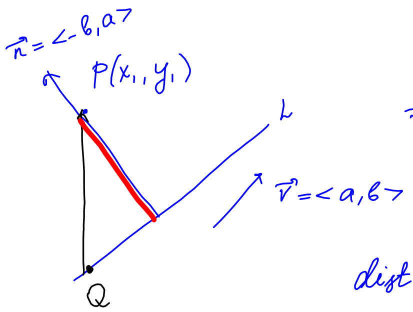


Find the distance from $P(0,3)$ to the line

$y-2x=9$

$-2x+y-9=0$

$$d=\left|\frac{-2(0)+3-9}{\sqrt{(-2)^2+1}}\right| = \left|-\frac{6}{\sqrt{5}}\right| = \boxed{\frac{6}{\sqrt{5}}}$$

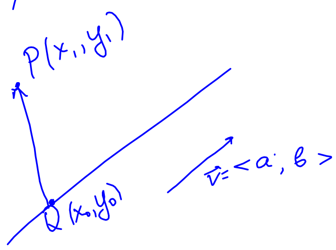


$\vec{v} \parallel L$

$\vec{n} \perp L$

$$dist = \left|comp_{\vec{n}} \vec{QP}\right| = \left|comp_{\vec{v}^{\perp}} \vec{QP}\right|$$

Find the distance from the point (x_1,y_1) to the line parallel to the vector $\langle a,b\rangle$ trough the Point (x_0,y_0)



$\vec{QP} = \langle x_1-x_0, y_1-y_0\rangle$

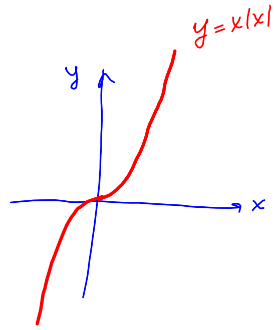
$$d=\left|comp_{\vec{v}^{\perp}} \vec{QP}\right| = \left|\frac{\langle x_1-x_0, y_1-y_0\rangle\cdot\langle -b,a\rangle}{\sqrt{a^2+b^2}}\right|$$

$x+1$

13. Let $f(x) = x|x|$

(a) For what values of x is f differentiable?

(b) Find a formula for f'.



$x|x| = \begin{cases} (x)x, & \text{if } x \geq 0 \\ -x(x) & \text{if } x < 0 \end{cases}$, $x|x| = \begin{cases} (x)^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

f is not differentiable @ $x=a$ if

- f has a corner @ $x=a$
- f is discontinuous @ $x=a$
- f has a vertical tangent @ $x=a$

differentiable on $(-\infty, \infty)$

(b) $x|x| = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

$\boxed{(x|x|)' = \begin{cases} 2x, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{cases}} = 2|x|$