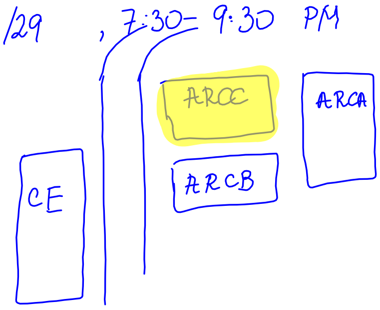
9/29, 7:30- 9:30 PM in ARCC 105



20 MC, 3.5 points each

4 free responce

no calculators!

scantron 882-E

(long green one)

score <70 points

retest 10/7 25 MC questions

1. $\lim\limits_{x\to+\infty} \dfrac{(3x-2)(x+5)}{(3-x)(1+2x)} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}\left(3-\cancel{\frac{2}{x}}^{\,0}\right)\left(1+\cancel{\frac{5}{x}}^{\,0}\right)}{\cancel{x^2}\left(\cancel{\frac{3}{x}}^{\,0}-1\right)\left(\cancel{\frac{1}{x}}^{\,0}+2\right)} = \lim\limits_{x\to\infty} \dfrac{(3)(1)}{(-2)} = -\dfrac{3}{2}$

(a) $-\frac{3}{2}$

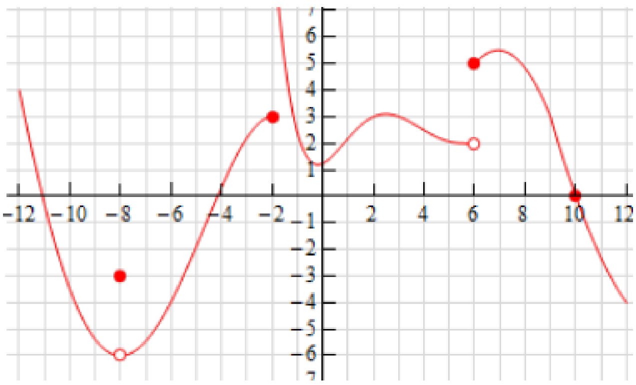
(b) 3

(c) $-\frac{5}{3}$

(d) 1

(e) $\frac{3}{2}$

2. Given the graph of $f(x)$ shown below, which of the following statements is true?



$\lim\limits_{x\to-2^-} f(x) = 3$

$\lim\limits_{x\to-2^+} f(x) = \infty$

$\lim\limits_{x\to-2} f(x)$ DNE

removable discontinuity @ -8

$\lim\limits_{x\to-8} f(x) = -6$

$f(-8) = -3$

(a) $f(x)$ is not continuous at $x=-8$ because $\lim\limits_{x\to-8} f(x)$ does not exist. not true

(b) $f(x)$ is not continuous at $x=-2$ because $\lim\limits_{x\to-2} f(x) = \infty$. not true

(c) $f(x)$ is not continuous at $x=6$ because $f(6)$ does not exist. not true, $f(6)=5$

(d) $f(x)$ is not continuous at $x=-8$ because $\lim\limits_{x\to-8} f(x) \neq f(-8)$. true

(e) All of the above statements are false.

vector equation

$\vec{r}(t) = \langle 4,-7\rangle + t\langle 3,4\rangle$

3. Which of the following is a set of parametric equations for the line passing through the point $(4,-7)$ and perpendicular to the line $3x+4y=-8$?

(a) $x = 4+3t,\ y=-7+4t$

(b) ~~$x=4+4t,\ y=-7+3t$~~

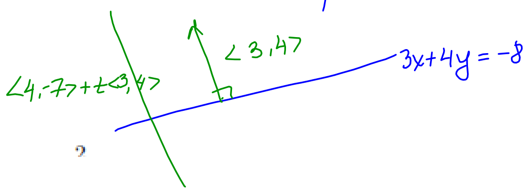
(c) ~~$x=4+3t,\ y=-7-4t$~~

(d) ~~$x=3t,\ y=-2+4t$~~

(e) ~~$x=4-4t,\ y=-7+3t$~~

$3x+4y=-8$ $\perp$ to the vector $\langle 3,4\rangle$

line should be parallel to $\langle 3,4\rangle$



line L: $Ax+By+C=0$, passes through (x_0,y_0)

line perpendicular to the line L is

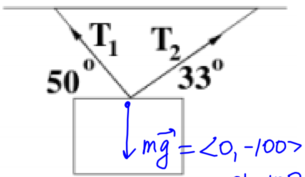
$\vec{r}(t) = \langle x_0,y_0\rangle + t\langle A,B\rangle$

if $x \to -\infty$, then $\sqrt{x^2} = -x$

4. $\displaystyle\lim_{x \to -\infty} \frac{-5x+2}{\sqrt{4x^2+x+4}} = \lim_{x\to-\infty} \frac{x\left(-5+\cancel{\frac{2}{x}}^{\,0}\right)}{\sqrt{x^2\left(4+\cancel{\frac{x}{x^2}}^{\,0}+\cancel{\frac{4}{x^2}}^{\,0}\right)}} = \lim_{x\to-\infty} \frac{-5x}{\sqrt{4x^2}} = \lim_{x\to-\infty} \frac{-5x}{-2x} = \frac{5}{2}$

   (a) $\frac{5}{2}$
   (b) $-\frac{5}{4}$
   (c) 5
   (d) $\frac{5}{4}$
   (e) None of the above

5. A 100 pound weight is suspended by two wires, $\mathbf{T_1}$ and $\mathbf{T_2}$, as shown below. Which of the following equations must be solved in order to find the magnitude of the tension in each wire?



$\vec{mg} = \langle 0, -100 \rangle$

$|\vec{mg}| = 100$

$\vec{T}_1 = |\vec{T}_1| \langle -\cos 50°, \sin 50° \rangle$

$\vec{T}_2 = |\vec{T}_2| \langle \cos 33°, \sin 33° \rangle$

$\vec{T}_1 + \vec{T}_2 + \vec{mg} = \vec{0}, \quad \vec{mg} = \langle 0, -100 \rangle$

$|\vec{T}_1|(-\cos 50°) + |\vec{T}_2| \cos 33° = 0$

$|\vec{T}_1| \sin 50° + |\vec{T}_2| \sin 33° = 100$

   (a) $|\mathbf{T_2}|\cos(33°) - |\mathbf{T_1}|\cos(50°) = 100$ and $|\mathbf{T_2}|\sin(33°) - |\mathbf{T_1}|\sin(50°) = 0$
   (b) $|\mathbf{T_2}|\cos(33°) + |\mathbf{T_1}|\cos(50°) = 100$ and $|\mathbf{T_2}|\sin(33°) + |\mathbf{T_1}|\sin(50°) = 0$
   (c) $|\mathbf{T_2}|\cos(33°) - |\mathbf{T_1}|\cos(50°) = 0$ and $|\mathbf{T_2}|\sin(33°) + |\mathbf{T_1}|\sin(50°) = 100$
   (d) $|\mathbf{T_2}|\cos(33°) + |\mathbf{T_1}|\cos(50°) = 0$ and $|\mathbf{T_2}|\sin(33°) + |\mathbf{T_1}|\sin(50°) = 100$
   (e) $|\mathbf{T_2}|\cos(33°) - |\mathbf{T_1}|\cos(50°) = 0$ and $|\mathbf{T_2}|\sin(33°) - |\mathbf{T_1}|\sin(50°) = 100$

6. Find the average rate of change of $f(x) = \tan x$ over the interval $\left[0, \frac{\pi}{4}\right]$.

$$\frac{\tan\frac{\pi}{4} - \tan 0}{\frac{\pi}{4} - 0} = \frac{1}{\frac{\pi}{4}} = \frac{4}{\pi}$$

   (a) 1
   (b) $\frac{4}{\pi}$
   (c) $\frac{\pi}{4}$
   (d) $\frac{2}{\pi}$
   (e) π

7. A man uses a *horizontal* force of 20 pounds on a box as he pushes it up a ramp that is 7 feet long and is inclined at an angle of 60° above the horizontal. Find the work done.

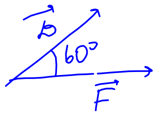
(a) 70 foot pounds

(b) 140 foot pounds

(c) $140\sqrt{3}$ foot pounds

(d) $70\sqrt{2}$ foot pounds

(e) $70\sqrt{3}$ foot pounds



$|\vec{F}| = 20$

$|\vec{D}| = 7$

$W = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos 60° = (20)(7)\left(\frac{1}{2}\right) = 70$

8. $\lim_{t\to 4} \dfrac{t^2 - 16}{\sqrt{t} - 2} =$

$\lim_{t\to 4} \dfrac{(t^2-16)(\sqrt{t}+2)}{(\sqrt{t}-2)(\sqrt{t}+2)} = \lim_{t\to 4} \dfrac{(t^2-16)(\sqrt{t}+2)}{t-4} = \lim_{t\to 4} \dfrac{\cancel{(t-4)}(t+4)(\sqrt{t}+2)}{\cancel{t-4}} = (4+4)(\sqrt{4}+2)$

$= (8)(4) = 32$

(a) 8

(b) 4

(c) 16

(d) 32

(e) The limit does not exist.

9. Consider the triangle with vertices $A(1,3)$, $B(2,1)$ and $C(-2,0)$. Find the angle, α, located at vertex A, that is $\angle BAC$.

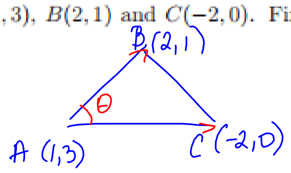
(a) $\alpha = \arccos\left(\dfrac{-1}{\sqrt{10}}\right)$

(b) $\alpha = \arccos\left(\dfrac{3}{\sqrt{10}}\right)$

(c) $\alpha = \arccos\left(\dfrac{-3}{\sqrt{30}}\right)$

(d) $\alpha = \arccos\left(\dfrac{1}{\sqrt{10}}\right)$

(e) $\alpha = \arccos\left(\dfrac{3}{\sqrt{30}}\right)$



$\vec{AB} = \langle 2-1, 1-3\rangle = \langle 1, -2\rangle$

$\vec{AC} = \langle -2-1, 0-3\rangle = \langle -3, -3\rangle$

$\cos(\angle BAC) = \dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\,|\vec{AC}|} = \dfrac{-3+6}{\sqrt{1+4}\sqrt{9+9}} = \dfrac{\cancel{3}}{\cancel{3}\sqrt{5}\cdot\sqrt{2}} = \dfrac{1}{\sqrt{10}}$

$\sqrt{18} = \sqrt{9\cdot 2} = 3\sqrt{2}$

10. Find the vector projection of $2\mathbf{i} - \mathbf{j}$ onto $3\mathbf{i} + \mathbf{j}$.

$\vec{m} = \langle 2, -1 \rangle$, $\vec{n} = \langle 3, 1 \rangle$

$\text{proj}_{\vec{n}} \vec{m} = \frac{\vec{m} \cdot \vec{n}}{|\vec{n}|^2} \vec{n} = \frac{\langle 2,-1 \rangle \cdot \langle 3,1 \rangle}{\left(\sqrt{3^2+1}\right)^2} \langle 3,1 \rangle$

$= \frac{6-1}{10} \langle 3,1 \rangle = \frac{5}{10} \langle 3,1 \rangle = \frac{1}{2} \langle 3,1 \rangle = \langle \frac{3}{2}, \frac{1}{2} \rangle$

(a) $3\mathbf{i} + \mathbf{j}$

(b) $\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$

(c) $\frac{15}{\sqrt{10}}\mathbf{i} + \frac{5}{\sqrt{10}}\mathbf{j}$

(d) $3\sqrt{5}\mathbf{i} + \sqrt{5}\mathbf{j}$

(e) $\frac{3}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$

$x = -7.99$

11. $\lim_{x \to -8^+} \frac{x-9}{x(x+8)} =$

$\frac{-7.99-9}{-7.99(-7.99+8)}$ $\frac{-}{-(+)} = +$

(a) ∞

(b) 0

(c) 1

(d) $-\infty$

(e) does not exist

$f'(x) = 2x - 4$

12. If $f(x) = x^2 - 4x + 1$, what is $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$? $= f'(3)$

$f'(3) = 2(3) - 4 = 2$

(a) 10

(b) 2

(c) 1

(d) 3

(e) 6

removable discontinuity @ $x=1$

13. Find the vertical asymptote(s) for the function $f(x) = \frac{x^3+2x^2-3x}{x^2+x-2}$ $= \frac{x(x^2+2x-3)}{(x+2)(x-1)} = \frac{x\cancel{(x-1)}(x+3)}{(x+2)\cancel{(x-1)}}$

(a) $x = 1$ and $x = -2$ only
(b) $x = -2$ only
(c) $x = -3$ and $x = 0$ only
(d) $x = -2$, $x = -3$ and $x = 0$ only
(e) $x = 1$ only

V.A. $x+2=0$ or $\boxed{x=-2}$

14. Suppose s and w are real numbers and let $\mathbf{a} = \langle 3s, 7\rangle$ and $\mathbf{b} = \langle -2, 5w\rangle$. If $\mathbf{a}$ is perpendicular to $\mathbf{b}$, what is the relationship between s and w?

$\vec{a}\cdot\vec{b}=0$

(a) $-6s + 35w = 1$
(b) $3s - 5w + 2 = 0$
(c) $-6s + 35w = 0$
(d) $-6s + 35w = 90$
(e) $3s + 5w + 11 = 0$

$3(s)(-2)+7(5w)=0$

$-6s+35w=0$

$x^3+2x^2+2=44$

15. Which of the following intervals contains a solution to the equation $x^3 + 2x^2 - 42 = 0$?

$f(x)= x^3+2x^2-42$

~~(a) $(-2, 0)$~~
~~(b) $(-1, 0)$~~
~~(c) $(0, 1)$~~
~~(d) $(1, 2)$~~
(e) $(2, 3)$

$f(-2) = (-2)^3+2(-2)^2-42<0$
$f(-1) = (-1)^3+2(-1)-42<0$
$f(0) = -42<0$
$f(1) = 1+2-42<0$
$f(2) = 8+8-42<0$
$f(3) = 27+2(9)-42>0$

PART II: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (i) (8 pts) If $f(x) = \frac{3}{4x-1}$, Using the **Limit Definition of the Derivative**, show that $f'(x) = \frac{-12}{(4x-1)^2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{3}{4x-1}, \quad f(x+h) = \frac{3}{4(x+h)-1} = \frac{3}{4x+4h-1}$$

$$f'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{3}{4x+4h-1} - \frac{3}{4x-1}\right] = \lim_{h\to 0} \frac{3}{h} \frac{4x-1-(4x+4h-1)}{(4x+4h-1)(4x-1)}$$

$$= \lim_{h\to 0} \frac{3}{h} \frac{\cancel{4x}-\cancel{1}-\cancel{4x}-4h+\cancel{1}}{(4x+4h-1)(4x-1)} = \lim_{h\to 0} \frac{-12\cancel{h}}{\cancel{h}(4x+4h^{0}-1)(4x-1)}$$

$$= \boxed{\frac{-12}{(4x-1)^2}}$$

(ii) (4 pts) Find the equation of the tangent line to the graph of $f(x)$ at $x = -2$.

$$|x-3| = \begin{cases} -(x-3), & x \le 3 \\ +(x-3), & x > 3 \end{cases}$$

17. Find the limit, if it exists. **If it does not exist, explain why**.

(i) (3 pts) $\lim\limits_{x\to 3^-} \dfrac{|x-3|}{x^2-4x+3}$ $= \lim\limits_{x\to 3} \dfrac{-(x-3)}{x^2-4x+3} = \lim\limits_{x\to 3} \dfrac{-\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = -\dfrac{1}{3-1} = \boxed{-\dfrac{1}{2}}$

$x<3$

(ii) (3 pts) $\lim\limits_{x\to 3^+} \dfrac{|x-3|}{x^2-4x+3}$ $= \lim\limits_{x\to 3} \dfrac{\cancel{x-3}}{\cancel{(x-3)}(x-1)} = \boxed{\dfrac{1}{2}}$

$x>3$

(iii) (2 pts) $\lim\limits_{x\to 3} \dfrac{|x-3|}{x^2-4x+3}$ DNE

18. Consider $f(x) = \begin{cases} ax^2 + 2x & \text{if } x < 2 \\ K & \text{if } x = 2 \\ x^3 + ax - 3 & \text{if } x > 2 \end{cases}$

(i) (2 pts) Find $\lim_{x\to 2^-} f(x)$ in terms of a.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2+2x) = a(2)^2+2(2) = \boxed{4a+4}$

(ii) (2 pts) Find $\lim_{x\to 2^+} f(x)$ in terms of a.

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} (x^3+ax-3) = 2^3 + a(2) - 3$

$= 8+2a-3 = \boxed{5+2a}$

(iii) (2 pts) For what value of a does $\lim_{x\to 2} f(x)$ exist?

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$4a+4 = 5+2a$

$2a = +1$, $\boxed{a = +\tfrac{1}{2}}$

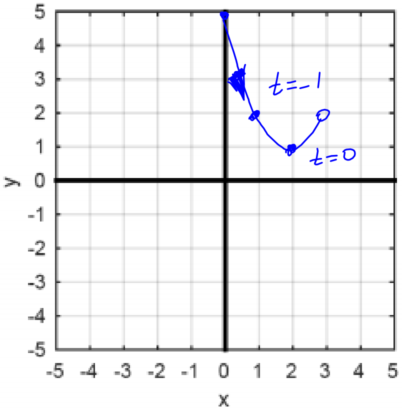
$\lim_{x\to 2} f(x) = 4\left(\frac{1}{2}\right)+4 = 6$

(iv) (2 pts) Using the value of a found above, for what value of K is $f(x)$ continuous?

$\underline{K = f(2)} = \underline{\lim_{x\to 2} f(x) = 6}$

$\boxed{K=6}$

19. (6 pts) Sketch the graph of $x = t + 2$, $y = t^2 + 1$, $-2 \le t < 1$ on the grid provided below.



$t = x - 2$

$y = (x-2)^2 + 1$

parabola

Domain

$-2 \le t < 1$

$-2+2 \le t+2 < 1+2$

$0 \le x < 3$

$t=0 \qquad x(0) = 2$

$y(0) = 1$

$t=-1 \qquad x(-1) = -1+2 = 1$

$y(-1) = (-1)^2+1 = 2$

20. (6 pts) Find the cartesian equation of the curve $x = 1 + \cos t$, $y = 3 + \sin t$, $0 \le t \le 2\pi$. Your answer must not be in terms of inverse trig functions.

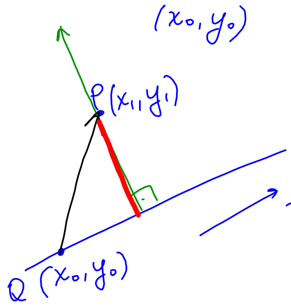
$\cos^2 t + \sin^2 t = 1$

$x = 1 + \cos t \longrightarrow \cos t = x - 1$

$y = 3 + \sin t \longrightarrow \sin t = y - 3$

$$\boxed{\underbrace{(x-1)^2}_{\cos^2 t} + \underbrace{(y-3)^2}_{\sin^2 t} = 1}$$

Find the distance from the point (x_1, y_1) to the line parallel to the vector $\vec{v} = \langle a, b \rangle$ through the point (x_0, y_0)



$\vec{v} = \langle a, b \rangle$

$\vec{v}^{\perp} = \langle -b, a \rangle$

$\overrightarrow{QP} = \langle x_1 - x_0, y_1 - y_0 \rangle$

$$d = \left| \text{comp}_{\vec{v}^{\perp}} \overrightarrow{QP} \right| = \left| \frac{\overrightarrow{QP} \cdot \vec{v}^{\perp}}{|\vec{v}^{\perp}|} \right|$$

$$= \frac{(x_1 - x_0)(-b) + (y_1 - y_0)a}{\sqrt{a^2 + b^2}}$$

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vector eqn. $\vec{r}(t) = \langle x_0, y_0 \rangle + t \langle a, b \rangle$

$x(t) = x_0 + ta \longrightarrow t = \dfrac{x - x_0}{a}$

$y(t) = y_0 + tb$

$$y = y_0 + \frac{x - x_0}{a} b$$

$$ay = ay_0 + (x - x_0) b$$

Cartesian eqn. $ay - xb - ay_0 + bx_0 = 0$ C

now can use the distance formula

(a) $\lim_{x\to 5} \frac{x^2 - 5x + 10}{x^2 - 25}$ $= \frac{5^2 - 5(5)+10}{5^2-25} = \frac{10}{0}$ $\boxed{DNE}$

$x = 5.01$

$\lim_{x\to 5^+} \frac{x^2-5x+10}{x^2-25} = \frac{5.01^2 - 5(5.01)+10}{(5.01)^2 - 25} = \frac{+}{+} = \infty$

$\lim_{x\to 5^-} \frac{x^2-5x+10}{x^2-25} = \frac{4.99^2 - 5(4.99)+10}{(4.99)^2-25} = \frac{+}{-} = -\infty$

$x = 4.99$

$$\lim_{y\to\infty}\frac{\boxed{7}y^3+4y}{\boxed{2}y^3-y^2+3} = \boxed{\frac{7}{2}}$$

the largest power of y on the top is $3 =$ the largest power of y on the bottom

18. Consider $f(x) = \begin{cases} x^2 + 6c - 5 & \text{if } x < 2 \\ 27 & \text{if } x = 2 \\ 2c - x + 9 & \text{if } x > 2 \end{cases}$

(i) (3 pts) Find $\lim_{x \to 2^-} f(x)$ in terms of c.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (x^2 + 6c - 5) = 4 + 6c - 5 = \boxed{6c - 1}$$

(ii) (3 pts) Find $\lim_{x \to 2^+} f(x)$ in terms of c.

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (2c - x + 9) = 2c - 2 + 9 = \boxed{2c + 7}$$

(iii) (4 pts) For what value of c does $\lim_{x \to 2} f(x)$ exist?

$$6c - 1 = 2c + 7$$
$$4c = 8 \Rightarrow \boxed{c = 2}$$

(iv) (3 pts) For the value of c found above, what is $\lim_{x \to 2} f(x)$?

$$\lim_{x \to 2} f(x) = 2(2) + 7 = \boxed{11}$$

(v) (3 pts) For the value of c above, is $f(x)$ continuous at $x = 2$? Support your answer.

$\lim_{x \to 2} f(x) = 11$ $\qquad$ $f(2) = 27$

f is discontinuous @ $x=2$ (removable discont.)

$\lim_{x \to 2} f(x) \neq f(2)$

(g) $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)\dfrac{\left(x-\sqrt{x^2+2x}\right)}{x-\sqrt{x^2+2x}} = \lim_{x\to-\infty}\dfrac{\cancel{x^2}-(\cancel{x^2}+2x)}{x-\sqrt{x^2(1+\cancel{\frac{2}{x}}^{\,0})}} = \lim_{x\to-\infty}\dfrac{-2x}{x-\sqrt{x^2}}$

$\sqrt{x^2}=-x$, if $x<0$

$= \lim_{x\to-\infty}\dfrac{-2x}{x-(-x)} = \lim_{x\to-\infty}\dfrac{-2x}{2x} = -1$

8. Given the parametric curve $x(t) = 1 + \cos t$, $y(t) = 1 - \sin^2 t$.

(a) Find a Cartesian equation for this curve.

(b) Does the parametric curve go through the point (1,0)? If yes, give the value(s) of t.

(c) Sketch the graph of the parametric curve on the interval $0 \le t \le \pi$, include the direction of the path.

Find the values for t such that $x(t)=1$ and $y(t)=0$.

$1+\cos t = 1$ and $1-\sin^2 t = 0$

$\cos t = 0$

$\boxed{t = \frac{\pi}{2} + \pi n}$ n is an arbitrary integer

$\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$

$0 \le x \le 3$

$\cos t = x - 1$

$\sin^2 t = 1 - y$

$\cos^2 t + \sin^2 t = 1$

$(x-1)^2 + 1 - y = 1$

$y = (x-1)^2$

Domain

$0 \le x \le \cancel{3}\, 2$

$x = \cos t + 1$

$-1 \le \cos t \le 1$

$0 \le \cos t + 1 \le 2$

$0 \le x \le 2$

$t=0$: $x(0) = 1+\cos 0 = 2$, $y(0) = 1 - \sin^2 0 = 1$

$t=\pi$: $x(\pi) = 1+\cos\pi = 1-1 = 0$, $y(\pi) = 1 - \sin^2\pi = 1$