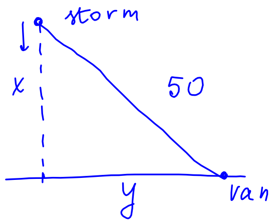
Ex16) A storm is 50 *miles* offshore and its path is perpendicular to a straight shoreline. It is approaching the shore at a rate of 4 *mph*. A van traveling along the shoreline wants to stay exactly 50 *miles* from the storm and remain along the shoreline. The van starts at the point on the shoreline in the path of the storm. Find the speed of the van when the storm is 40 *miles* from the shore.

$\frac{d}{dt}$



$\frac{dx}{dt} = -4 \text{ mph}$

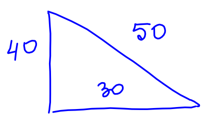
$\frac{dy}{dx}$ - ? when $x = 40$

$x^2 + y^2 = 50^2$

$\frac{d}{dt}(y^2) = \frac{d}{dt}(2500 - x^2)$

$2y \frac{dy}{dt} = -2x \frac{dx}{dt}$

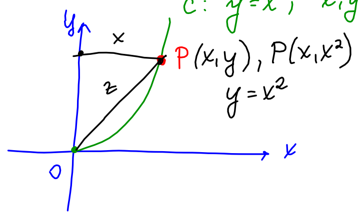
$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{40}{30}(-4) = \boxed{\frac{+16}{3} \text{ mph}}$



$y = \sqrt{50^2 - 40^2} = 30$

15. (**12 points**) Let $O(0,0)$ denote the origin of the axis of co-ordinates, and let C denote the portion of the parabola $y = x^2$ which lies in the first quadrant. A particle P begins to move from O, along the curve C, in such a way that its distance from the y-axis is increasing at a rate of 3 units per second. How fast is the **square of the distance** between P and O increasing, at the instant when P crosses the line $y = 4$?

$C: \ y = x^2, \ x, y \geq 0.$



$\frac{dx}{dt} = 3$

z is the distance between P and O

$\frac{d(z^2)}{dt}$ when $y = 4$ $(x^2 = 4)$ or $(x = 2)$

$$z^2 = x^2 + (x^2)^2 = x^2 + x^4$$

$$\frac{d(z^2)}{dt} = 2x\frac{dx}{dt} + 4x^3\frac{dx}{dt}$$

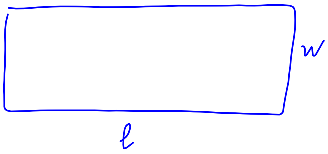
$$= \frac{dx}{dt}\left(2x + 4x^3\right)$$

$$= 3\left(2(2) + 4(2^3)\right)$$

$$= 3(4 + 32) = 3(36) = \boxed{108 \text{ units}^2/\text{sec}}$$

The length of a rectangle is increasing at a rate of 5 cm/sec and its width is decreasing at a rate of 4 cm/sec. When the length is 10 cm and the width is 20 cm, what is the rate of change of the area of the rectangle?

(a) 20 cm^2/sec



$\frac{d\ell}{dt} = 5, \quad \frac{dw}{dt} = -4$

Find $\frac{dA}{dt}$, when $\ell = 10$, $w = 20$

$$\frac{dA}{dt} = \frac{d}{dt}(\ell w)$$

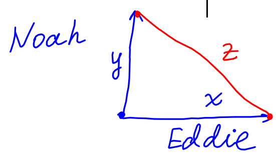
$$\frac{dA}{dt} = \frac{d\ell}{dt} w + \ell \cdot \frac{dw}{dt}$$

$$= 5(20) - 4(10) = \boxed{60 \ \text{cm}^2/\text{sec}}$$

2. (8 points) Noah travels due north and Eddie travels due east from a common starting point. At time t (in seconds), Noah's distance(in feet) from the starting location is y and Eddie's distance from the starting location is x. At what rate is the distance between Noah and Eddie changing after 2 seconds?

$y = 10 + 4t + \frac{1}{2}t^2$; $\frac{dy}{dt} = 4 + t$

$x = 7 + 4t$; $\frac{dx}{dt} = 4$



$x(2) = 7 + 4(2) = 15$

$y(2) = 10 + 4(2) + \frac{1}{2}(2)^2 = 20$

$z(2) = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$

$\frac{dx}{dt}(2) = 4$

$\frac{dy}{dt}(2) = 4 + 2 = 6$

$\frac{d}{dt} z^2 = \frac{d}{dt}(x^2 + y^2)$

Find $\frac{dz}{dt}$ after 2 sec.

$\cancel{2}z \frac{dz}{dt} = \cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt}$

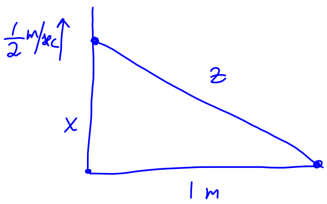
$\frac{dz}{dt} = \frac{1}{z}\left(x \frac{dx}{dt} + y \frac{dy}{dt}\right)$

$\frac{dz}{dt} = \boxed{\frac{1}{25}\left(15(4) + 20(6)\right)}$

$= \frac{1}{5}(12 + 24) = \boxed{\frac{36}{5} \text{ ft/s}}$

A dog on a chain sits 1 meter from a telephone pole. A squirrel begins running up the pole at a speed of $\frac{1}{2}$ meter per second. How fast is the distance changing (in m/s) between the squirrel and the spot the dog is sitting on when the squirrel has traveled 3 meters up the pole?

(a) None of these



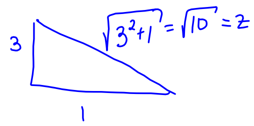
$\frac{dz}{dt}$ when $x = 3$

$$z^2 = x^2 + 1$$

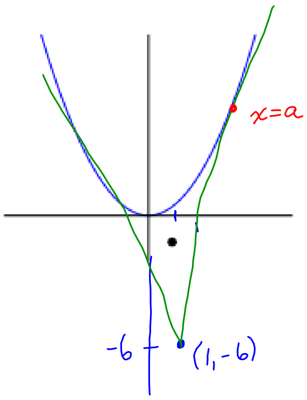
$$2z\frac{dz}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$$

$$= \frac{3}{\sqrt{10}}\,\frac{1}{2} = \boxed{\frac{3}{2\sqrt{10}} \text{ m/s}}$$



18. a) (3 pts) Show by means of a sketch that there are two lines tangent to the parabola $y = 2x^2$ that pass through the point $(1, -6)$.



b) (8 pts) Find an equation of each of these tangent lines.

through (1,-6)

$y = 2x^2$

Find the equation of the tangent line to $y = 2x^2$ at the point where $x = a$.

$y = f'(a)(x-a) + f(a)$, $f(x) = 2x^2$, $f(a) = 2a^2$

$f'(x) = 4x$, $f'(a) = 4a$

$y = 4a(x-a) + 2a^2$

$\boxed{y = 4ax - 2a^2}$ passes through $(1, -6)$

$-6 = 4a - 2a^2$

$2a^2 - 4a - 6 = 0$

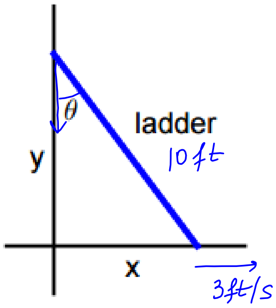
$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

$a_1 = 3$, $a_2 = -1$

$a = 3$	$a = -1$
$y = 4(3)x - 2(3)^2$	$y = 4(-1)x - 2(-1)^2$
$\boxed{y = 12x - 18}$	$\boxed{y = -4x - 2}$

16. A ladder 10 feet long is leaning against a vertical wall. The base of the ladder is pulled away from the wall at a rate of 3 ft/s.

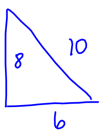


$\frac{dx}{dt} = 3$

a) (6 pts) How fast is the top of the ladder sliding down the wall when the base is 6 feet from the wall?

$\frac{dy}{dt}$ when $x = 6$

$\sqrt{100-36} = \sqrt{64} = 8$

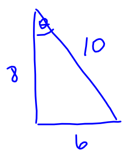


$y^2 = 10^2 - x^2$

$2y \frac{dy}{dt} = -2x \frac{dx}{dt}$

$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{6}{8}(3) = \boxed{-\frac{9}{4} \text{ ft/s}}$

b) (5 pts) Find the rate at which angle betw~~x~~en the ladder and the wall is changing when the base of the ladder is 6 feet from the wall.



$\cos\theta = \frac{8}{10} = \frac{4}{5}$

$\sin\theta = \frac{x}{10}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$

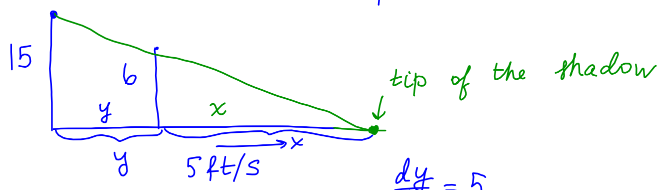
$\frac{d\theta}{dt} = \frac{1}{10\cos\theta} \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{10 \cdot \frac{8}{10}} \cdot 3 = \boxed{\frac{3}{8} \text{ rad/s}}$

Example 2. A street light is at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path.

(a.) How fast is the tip of his shadow moving when he is 40 ft from the pole?

$x+y$ is the distance from the tip of the shadow to the pole



$$\frac{dy}{dt} = 5$$

$$\frac{15}{6} = \frac{x+y}{x}$$

$$\frac{5}{2} = \frac{x+y}{x}; \quad 5x = 2x + 2y$$

$$3x = 2y$$

$$x = \frac{2}{3}y.$$

x is the length of the shadow

y is the distance between the man and the pole

$$\frac{d(x+y)}{dt} = \frac{d}{dt}\left(y + \frac{2}{3}y\right)$$

$$= \frac{5}{3}\frac{dy}{dt} = \boxed{\frac{25}{3} \text{ ft/s}}$$

(b.) How fast is his shadow lengthening at that point?

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{2}{3}y\right) = \frac{2}{3}\frac{dy}{dt} = \frac{2}{3}(5) = \boxed{\frac{10}{3}(\text{ft/s})}$$