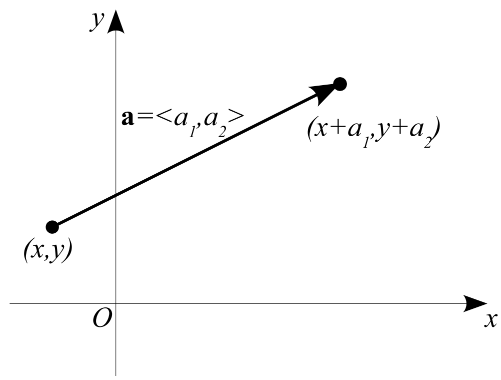
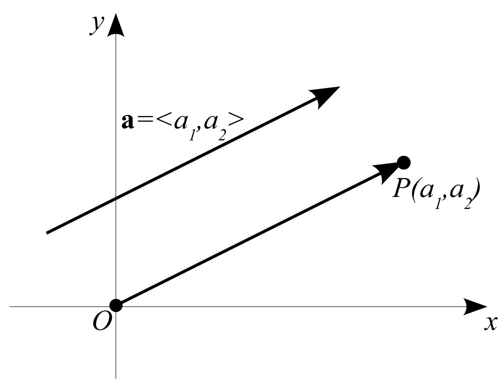
Chapter 1. **Introduction to vectors and vector functions**
Section 1.1 **Vectors**

Definition. A **two-dimensional vector** is an ordered pair $\vec{a} =< a_1, a_2 >$ of real numbers. The numbers a_1 and a_2 are called the **components** of $\vec{a}$.

A **representation** of the vector $\vec{a} =< a_1, a_2 >$ is a directed line segment $\overrightarrow{AB}$ from any point $A(x, y)$ to the point $B(x + a_1, y + a_2)$.



A particular representation of $\vec{a} =< a_1, a_2 >$ is the directed line segment $\overrightarrow{OP}$ from the origin to the point $P(a_1, a_2)$, and $\vec{a} =< a_1, a_2 >$ is called the **position vector** of the point $P(a_1, a_2)$.



Given the points $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$\overrightarrow{AB} =< x_2 - x_1, y_2 - y_1 >$$

Example 1. Find a vector $\vec{a}$ with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

(a) $A(1, 2)$, $B(3, 3)$;

(b) $A(1,-2)$, $B(-2,3)$.

The **magnitude (length)** $|\vec{a}|$ of $\vec{a}$ is the length of any its representation.
The length of $\vec{a} =< a_1, a_2 >$ is

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$$

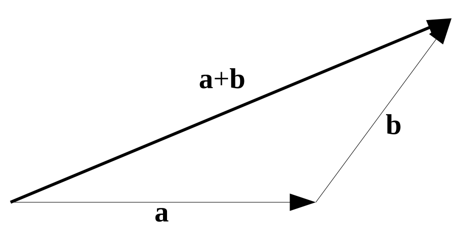
The length of the vector $\overrightarrow{AB}$ from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

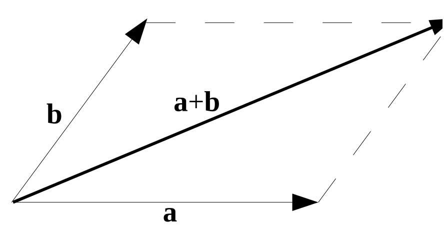
The only vector with length 0 is the **zero vector** $\vec{0} =< 0, 0 >$. This vector is the only vector with no specific direction.

Example 2. Find the length of the vectors from Example 1.

Vector addition If $\vec{a} =< a_1, a_2 >$ and $\vec{b} =< b_1, b_2 >$, then the vector $\vec{a} + \vec{b}$ is defined by $\vec{a} + \vec{b} =< a_1 + b_1, a_2 + b_2 >$.



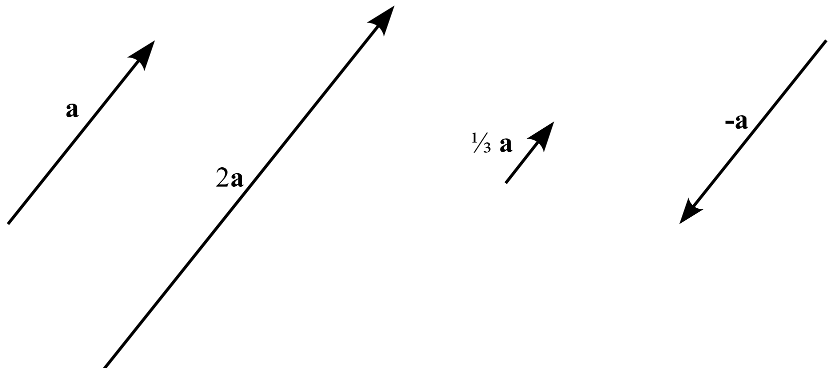
Triangle Law



Parallelogram Law

Multiplication of a vector by a scalar If c is a scalar and $\vec{a} = < a_1, a_2 >$, then the vector is defined by

$$c\vec{a} = < ca_1, ca_2 >$$



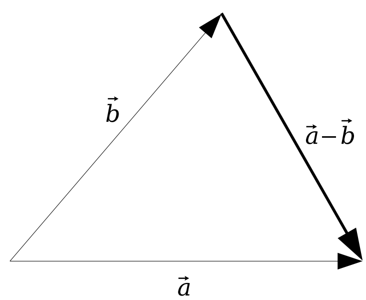
$$|c\vec{a}| = c|\vec{a}|$$

Two vectors $\vec{a}$ and $\vec{b}$ are called **parallel** if $\vec{b} = c\vec{a}$ for some scalar c.

By the **difference** of two vectors, we mean

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

so, if $\vec{a} = < a_1, a_2 >$ and $\vec{b} = < b_1, b_2 >$, then $\vec{a} - \vec{b} = < a_1 - b_1, a_2 - b_2 >$.



Example 3. If $\vec{a} = < -1, 2 >$ and $\vec{b} = < -2, -1 >$, find
(a) $\vec{a} + \vec{b}$

(b) $1/2\vec{b}$

(c) $\vec{a} - \vec{b}$

(d) $|2\vec{a} - 5\vec{b}|$

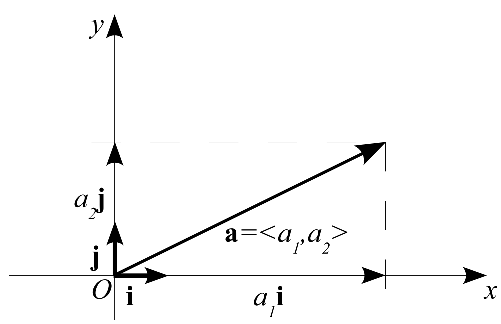
Properties of vectors. If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors and k and m are scalars, then

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
3. $\vec{a} + \vec{0} = \vec{a}$
4. $\vec{a} + (-\vec{a}) = \vec{0}$
5. $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$
6. $(k + m)\vec{a} = k\vec{a} + m\vec{a}$
7. $(km)\vec{a} = k(m\vec{a})$
8. $1\vec{a} = \vec{a}$

Let $\vec{\imath} =< 1, 0 >$ and $\vec{\jmath} =< 0, 1 >$.

$$|\vec{\imath}| = |\vec{\jmath}| = 1$$

$\vec{a} =< a_1, a_2 >= a_1\vec{\imath} + a_2\vec{\jmath}$



Example 4. Express $\vec{a} =< 2, 4 >$, $\vec{b} =< -1, 3 >$, and $2\vec{a} + \vec{b}$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.

A **unit vector** is a vector whose length is 1.

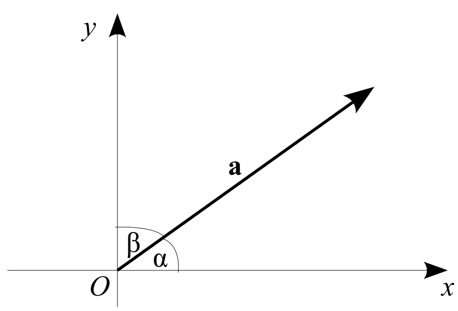
A vector

$$\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \left\langle \frac{a_1}{|\vec{a}|}, \frac{a_2}{|\vec{a}|} \right\rangle$$

is a unit vector that has the same direction as $\vec{a} =< a_1, a_2 >$.

Example 5. Given vectors $\vec{a} = \vec{\imath} - 2\vec{\jmath}$, $\vec{b} =< -2, 3 >$. Find a unit vector $\vec{u}$ that has the same direction as $2\vec{b} + \vec{a}$.

Direction angles and direction cosines. The **direction angles** of a nonzero vector $\vec{a}$ are the angles α and β in the interval $[0, \pi]$ that $\vec{a}$ makes with the positive $x-$ and $y-$ axes. The cosines of these direction angles, $\cos\alpha$ and $\cos\beta$ are called the **direction cosines** of the vector $\vec{a}$.



$$\cos\alpha = \frac{a_1}{|\vec{a}|}, \quad \cos\beta = \frac{a_2}{|\vec{a}|}, \qquad \cos^2\alpha + \cos^2\beta = 1$$

We can write

$$\vec{a} =< a_1, a_2 >= |\vec{a}| < \cos\alpha, \cos\beta >$$

Therefore

$$\frac{1}{|\vec{a}|}\vec{a} =< \cos\alpha, \cos\beta >$$

which says that the direction cosines of $\vec{a}$ are the components of the unit vector in the direction of $\vec{a}$.

Example 6. Let $\vec{c}$ be the vector obtained by rotating $\vec{a} =< 1, 3 >$ by an angle of 60 degrees in the counterclockwise direction. Compute the vector $\vec{c}$.

Example 7. Two forces $\vec{F_1}$ and $\vec{F_2}$ with magnitudes 10 lb and 12 lb act on an object at a point P as shown in the figure. Find the resultant force $\vec{F}$ acting at P as well as its magnitude and its direction.