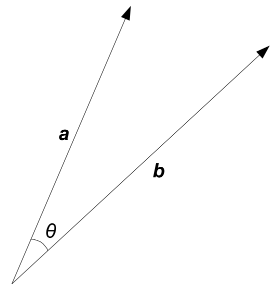
Section 1.2 **The dot product**

Definition. The **dot** or **scalar product** of two nonzero vectors $\vec{a}$ and $\vec{b}$ is the number

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$. If either $\vec{a}$ or $\vec{b}$ is $\vec{0}$, we define $\vec{a} \cdot \vec{b} = 0$.



Example 1. If the vectors $\vec{a}$ and $\vec{b}$ have lengths 2 and 6, and the angle between them is $\pi/4$, find $\vec{a} \cdot \vec{b}$.

If $\vec{a} =< a_1, a_2 >$ and $\vec{b} =< b_1, b_2 >$, then

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

Example 2. Find $\vec{a} \cdot \vec{b}$ if $\vec{a} =< 2, 3 >$ and $\vec{b} = \vec{\imath} - 3\vec{\jmath}$

Two nonzero vectors $\vec{a}$ and $\vec{b}$ are called **perpendicular** or **orthogonal** if the angle between them is $\pi/2$.

Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$.

Example 3. Find the angle between the vectors $\vec{a} = 6\vec{\imath} - 2\vec{\jmath}$ and $\vec{b} =< 1, 1 >$.

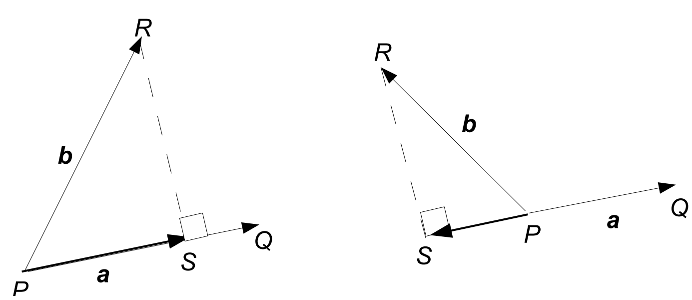
Example 4. Determine whether the given vectors are orthogonal, parallel, or neither.

(a.) $\vec{a} =< 1, -2 >$, $\vec{b} = -2\vec{\imath} + 4\vec{\jmath}$

(b.) $\vec{a} =< 3, 1 >$, $\vec{b} =< -3, 9 >$

(c.) $\vec{a} = -\vec{\imath} + 4\vec{\jmath}$, $\vec{b} = 3\vec{\imath} - 2\vec{\jmath}$

Properties of the dot product If $\vec{a}$, $\vec{b}$, an $\vec{c}$ are vectors and k is a scalar, then
1. $\vec{a} \cdot \vec{a} = |a|^2$
2. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
3. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
4. $(k\vec{a}) \cdot \vec{b} = k(\vec{a} \cdot \vec{b}) = \vec{a}(k\vec{b})$
5. $\vec{0} \cdot \vec{a} = 0$



$\vec{PS}$ =proj$_{\vec{a}}\vec{b}$ is called the **vector projection of $\vec{b}$ onto $\vec{a}$.**
$|\vec{PS}|$ =comp$_{\vec{a}}\vec{b}$ is called the **scalar projection of $\vec{b}$ onto** $\vec{a}$ or the **component of $\vec{b}$ along** $\vec{a}$.

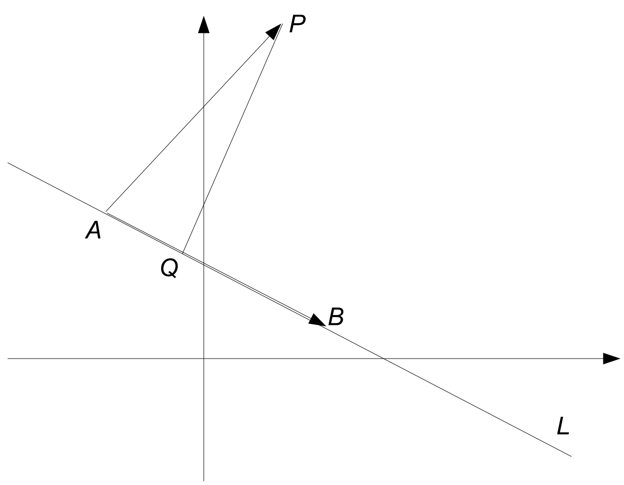
$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$$

Example 5. Find the scalar and the vector projections of $\vec{b} =< 4, 2 >$ onto $\vec{a} = \vec{\imath} + \vec{\jmath}$.

Definition. Given the nonzero vector $\vec{a} =< a_1, a_2 >$, the **orthogonal complement** of $\vec{a}$ is the vector $\vec{a}^{\perp} =< -a_2, a_1 >$.

Vectors $\vec{a}$ and $\vec{a}^{\perp}$ are orthogonal and $|\vec{a}| = |\vec{a}^{\perp}|$



The distance from the point P to the line L

$$|PQ| = \text{comp}_{\overrightarrow{AB}^{\perp}}\overrightarrow{AP}$$

Example 6. Find the distance from the point (0,4) to the line $2x + 5y = -3$.