

Section 3.10 **Related rates**

Strategy

1. Read the problem carefully.
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and the required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary, use the geometry of the situation to eliminate one of the variables by substitution.
6. Use the Chain Rule to differentiate both sides of the equation with respect to t .
7. Substitute the given information into the resulting equation and solve for the unknown rate.

Example 1. A spherical snowball is melting in such a way that its volume is decreasing at a rate of $1 \text{ cm}^3/\text{min}$. At what rate is the diameter decreasing when the diameter is 10 cm.

Example 2. A street light is at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path.

(a.) How fast is the tip of his shadow moving when he is 40 ft from the pole?

(b.) How fast is his shadow lengthening at that point?

Example 3. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

Example 4. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

Example 5. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm²/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm²?

Example 6. Water is leaking out of an inverted conical tank at a rate of $10,000 \text{ cm}^3/\text{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of $20 \text{ cm}/\text{min}$ when the height of the water is 2m, find the rate at which water is being pumped into the tank.