Section 3.4 **Derivatives of trigonometric functions**

Theorem. $\lim_{\theta \to 0} \sin\theta = 0.$

Theorem. $\lim_{\theta \to 0} \cos\theta = 1.$

Theorem. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

Corollary. $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0.$

Example 1. Find each limit

(a.) $\lim_{x \to 0} \frac{1 - \cos 2x}{2x^2}$

(b.) $\lim_{x \to 0} \frac{\sin 5x}{2x}$

(c.) $\lim_{x \to 0} \frac{\sin 4x}{\sin 3x}$

(d.) $\lim_{x \to 0} \frac{\tan 3x}{\sin 2x}$

Derivatives

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

Example 2. Find $\frac{dy}{dx}$

(a.) $y = \cos x - 2 \tan x$

(b.) $y = 2x(\sqrt{x} - \cot x)$

(c.) $y = x \csc x$

Example 3. Find the points on the curve $y = \dfrac{\cos x}{2 + \sin x}$ at which the tangent is horizontal.