Section 3.7. **Derivatives of vector functions**

Definition. The **derivative of a vector function** $\vec{r}(t)$ **at a number** a, denoted by $\vec{r}\,'(a)$, is

$$\vec{r}\,'(a) = \lim_{h\to 0} \frac{\vec{r}(a+h) - \vec{r}(a)}{h} = \lim_{t\to a} \frac{\vec{r}(t) - \vec{r}(a)}{t-a}$$

if the limits exist.

If $\vec{r}(t) =< x(t), y(t) >$ is a vector function, then

$$\vec{r}\,'(t) = \lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \left\langle \lim_{h\to 0} \frac{x(t+h) - x(t)}{h}, \lim_{h\to 0} \frac{y(t+h) - y(t)}{h} \right\rangle =< x'(t), y'(t) >$$

if both $x'(t)$ and $y'(t)$ exist.

Example 1. Find the domain and the derivative of the vector function $\vec{r}(t) =< t^2-4, \sqrt{t-4} >$.

Example 2. Find a tangent vector of unit length for the line given by $\vec{r}(t) = 2\sin t\vec{\imath} + 3\cos t\vec{\jmath}$ at the point where $t = \pi/6$.

Definition. If $\vec{r}(t) =< x(t), y(t) >$ is a vector function representing the position of a particle at time t, then
velocity at time t is

$$\vec{v}(t) = \vec{r}'(t) =< x'(t), y'(t) >$$

speed at time t is

$$s = |\vec{v}(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$$

Example 3. The vector function $\vec{r}(t) =< t, 25t - 5t^2 >$ represents the position of a particle at time t. Find the velocity and the speed at $t = 1$.