Section 3.8 **Higher derivatives**

$$\frac{d^2 f}{dx^2} = f''(x) = [f'(x)]'$$
$$\frac{d^3 f}{dx^3} = f'''(x) = [f''(x)]'$$
$$\frac{d^n f}{dx^n} = f^{(n)}(x) = [f^{(n-1)}(x)]'$$

Example 1. Find $f''(x)$ for the function $f(x) = \tan^3(2x - 1)$.

Example 2. Find $\frac{d^3}{dx^3}\left(\frac{1-x}{1+x}\right)$

Example 3. Find a formula for $f^{(n)}(x)$ for the following functions:
(a.) $f(x) = x^4 - 3x^3 + 16x$

(b.) $f(x) = \sqrt{x}$

(c.) $f(x) = x^n$

(c.) $f(x) = \dfrac{1}{(1-x)^2}$

Example 4. Find $f^{(25)}(x)$ if $f(x) = x \sin x$.

Acceleration.

Let $s = s(t)$ be the position function of an object that moves in a straight line.

The instantaneous rate of change of velocity with respect to time is called **acceleration** $a(t)$ of the object. Thus, the acceleration function is the derivative of the velocity function. Therefore

$$a(t) = v'(t) = s''(t).$$

Example 5. The equation of motion of a particle is $s(t) = 2t^3 - 7t^2 + 4t + 1$, where s is measured in meters and t in seconds. Find the acceleration as a function of time. What is acceleration after 1 s?

For the vector function $\vec{r}(t) =< x(t), y(t) >$

$$\vec{r}\,'(t) =< x'(t), y'(t) >$$

$$\vec{r}\,''(t) = [\vec{r}\,'(t)]' =< x''(t), y''(t) >$$

If $\vec{r}(t) =< x(t), y(t) >$ represents the position of an object then the acceleration vector is

$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) =< x''(t), y''(t) >$$

Example 5. Find the acceleration at $t = 2$ if $\vec{r}(t) = \sqrt{t^2 + 5}\vec{\imath} + t\vec{\jmath}$.

Implicit second derivative.

Example 6. Find $\frac{d^2y}{dx^2}$ if $x^2 + 6xy + y^2 = 8$.