RETEST 10/7, 7-9 PM BLOC 102

Bring the gray scantron sheet

Section 3.7. **Derivatives of vector functions**

Definition. The derivative of a vector function $\vec{r}(t)$ at a number a, denoted by $\vec{r}\,'(a)$, is

$$\boxed{\vec{r}\,'(a) = \lim_{h\to 0}\frac{\vec{r}(a+h)-\vec{r}(a)}{h} = \lim_{t\to a}\frac{\vec{r}(t)-\vec{r}(a)}{t-a}}$$

if the limits exist.

If $\vec{r}(t) =< x(t), y(t) >$ is a vector function, then

$$\vec{r}\,'(t) = \lim_{h\to 0}\frac{\vec{r}(t+h)-\vec{r}(t)}{h} = \left\langle \lim_{h\to 0}\frac{x(t+h)-x(t)}{h}, \lim_{h\to 0}\frac{y(t+h)-y(t)}{h}\right\rangle = \boxed{< x'(t), y'(t) > = \vec{r}\,'(t)}$$

if both $x'(t)$ and $y'(t)$ exist.

Example 1. Find the domain and the derivative of the vector function $\vec{r}(t) =< t^2-4, \sqrt{t-4} >$ $= < t^2-4, (t-4)^{1/2} >$

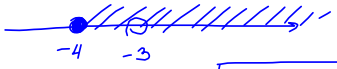
domain: $t-4 \geq 0 \rightarrow \boxed{t \geq 4}$

$\vec{r}\,'(t) = < (t^2-4)', [(t-4)^{1/2}]' >$

$= \boxed{< 2t, \frac{1}{2}(t-4)^{-1/2} >}$

Find the domain of the function $\vec{r}(t) = < \frac{1}{t+3}, \sqrt{t+4} >$

$t \neq -3$, $t \geq -4$



$\boxed{[-4,-3)\cup(-3,\infty)}$

Example 2. Find a tangent vector of unit length for the curve given by $\vec{r}(t) = 2\sin t\vec{i} + 3\cos t\vec{j}$ at the point where $t = \pi/6$.

$\vec{r}(t) = < 2\sin t, 3\cos t >$

tangent vector $\vec{v}(t) = \vec{r}\,'(t) = < (2\sin t)', (3\cos t)' >$

$= < 2\cos t, -3\sin t >$

$\vec{v}(\frac{\pi}{6}) = < 2\cos\frac{\pi}{6}, -3\sin\frac{\pi}{6} >$

$= < 2\frac{\sqrt{3}}{2}, -\frac{3}{2} > = < \sqrt{3}, -\frac{3}{2} >$

$|\vec{v}(\frac{\pi}{6})| = \sqrt{3+\frac{9}{4}} = \frac{\sqrt{21}}{2}$

$\vec{u} = \frac{\vec{v}(\frac{\pi}{6})}{|\vec{v}(\frac{\pi}{6})|} = \frac{< \sqrt{3}, -\frac{3}{2} >}{\frac{\sqrt{21}}{2}} = \frac{2}{\sqrt{21}} < \sqrt{3}, -\frac{3}{2} > = < \frac{2\sqrt{3}}{\sqrt{21}}, \frac{-3}{\sqrt{21}} >$

$= \boxed{< \frac{2}{\sqrt{7}}, -\frac{3}{\sqrt{21}} >}$

Definition. If $\vec{r}(t) =< x(t), y(t) >$ is a vector function representing the position of a particle at time t, then

velocity at time t is

$$\boxed{\vec{v}(t) = \vec{r}'(t) =< x'(t), y'(t) >}$$

speed at time t is

$$\boxed{s = |\vec{v}(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}}$$

Example 3. The vector function $\vec{r}(t) =< t, 25t - 5t^2 >$ represents the position of a particle at time t. Find the velocity and the speed at $t = 1$.

$\vec{v}(t) = \vec{r}'(t) = < t', (25t - 5t^2)' >$

$= < 1, 25 - 10t >$

$\vec{v}(1) = < 1, 25 - 10 > = \boxed{< 1, 15 >}$

$s = |\vec{v}(1)| = \sqrt{1 + 15^2} = \sqrt{1 + 225} = \boxed{\sqrt{226}}$