Section 4.3 **Logarithmic functions**

$\log_a x = y \iff a^y = x$

The cancellation equations

$$\log_a a^x = x \qquad a^{\log_a x} = x$$

If $a = e$, then

$$\log_e x = \ln x$$

Example 1. Evaluate:

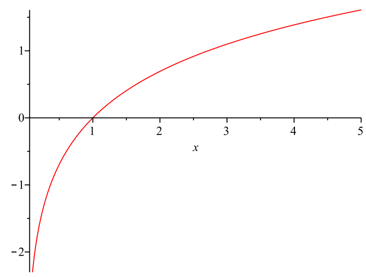
1. $\log_2 64$

2. $\log_6 \frac{1}{36}$

3. $2^{\log_2 3 + \log_2 5}$

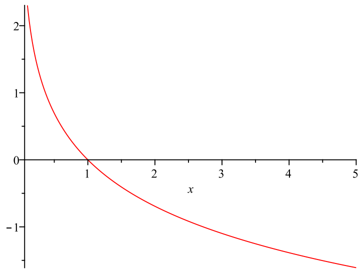
Theorem. Function $f(x) = \log_a x$ is one-to-one continuous function with domain $(0, \infty)$ and range $(-\infty, \infty)$.

1. If $a > 1$, then



- $f(x) = \log_a x$ is increasing function
- $\lim_{x \to \infty} \log_a x = \infty$
- $\lim_{x \to 0^+} \log_a x = -\infty$

2. If $0 < a < 1$, then



- $f(x) = \log_a x$ is decreasing function
- $\lim_{x \to \infty} \log_a x = -\infty$
- $\lim_{x \to 0^+} \log_a x = \infty$

Example 2. Find the domain and the range for the function $f(x) = \sqrt{x}\ln(x^2 - 1)$.

Example 3. Find the limit:

1. $\lim_{x \to 5^+} \ln(x - 5)$

2. $\lim_{x \to \infty} \log_2(x^2 - x)$

3. $\lim_{x \to \infty} \frac{\ln x}{1 + \ln x}$

Properties of logatithmic functions
If $x, y > 0$ and k is a constant, then

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
3. $\log_a x^k = k\ \log_a x \qquad \log_a \dfrac{1}{x} = -\log_a x$
4. $\log_{a^k} x = \dfrac{1}{k} \log_a x \qquad \log_{\frac{1}{a}} x = -\log_a x$
5. $\log_a a = 1$
6. $\log_a 1 = 0$
7. $\log_a x = \dfrac{\log_b x}{\log_b a}$

Example 4. Evaluate $\mathrm{e}^{3\ln 2 - 1} \ln(5\mathrm{e}^2)$

Example 5. Express the given quatities as a single logarithm:

1. $\log_2 x + 5\log_2(x+1) + \dfrac{1}{2}\log_2(x-1)$

2. $2\ln 4 - \ln 2$

3. $\log_8 a - \log_4 b + \log_2 c$

Example 6. Solve the equation:

1. $10(1+\mathrm{e}^{-x})^{-1} = 3$

2. $\log_2(2x+1) = 2 - \log_2(4x)$