

Section 4.5 Exponential growth and decay

If $y(t)$ is the value of a quantity y at time t and if the rate of change of y with respect to t is proportional to $y(t)$ at any time, then

$$\frac{dy}{dt} = ky$$

where k is a constant. This equation is called the **law of natural growth** if $k > 0$ or the **law of natural decay** if $k < 0$.

The only solution to this equation is

$$y(t) = y(0)e^{kt}$$

Example 1. A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria.

1. Find an expression for the number of bacteria after t hours.

2. Find the number of bacteria after 4 hours.

3. When will the population reach 30,000?

Example 2. Polonium-214 has a half-life of 1.4×10^{-4} s.

1. If a sample has a mass of 50 mg, find a formula for the mass that remains after t seconds.

2. Find the mass that remains after a hundredth of a second.

3. How long would it take for the mass to decay to 40 mg?

Example 3. A roast turkey is taken from the oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F .

1. If the temperature of turkey is 150°F after half an hour, what is the temperature after 45 min?

2. When will the turkey have cooled to 100°F ?

Example 4. A tank contains 1500 L of brine with a concentration of 0.3 kg of salt per liter. In order to dilute the solution, pure water is run into the tank at a rate of 20 L/min and the resulting solution, which is stirred continuously, runs out at the same rate.

1. How many kilograms of salt will remain after 30 min?

2. When will the concentration of salt be reduced to 0.2 kg/L?