Chapter 6. **Integrals**

Section 6.1 **Sigma notation**

Definition. If a_m, a_{m+1}, a_{m+2},...,a_n are real numbers and m and n are integers such that $m \leq n$, then

$$a_m + a_{m+1} + a_{m+2} + ... + a_n = \sum_{i=m}^{n} a_i$$

Example 1. Write the sum in sigma notation.

1. 1+2+3+...+10

2. 1+2+4+8+16+32

3. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}$

4. $\frac{3}{7} + \frac{4}{8} + \frac{5}{9} + \frac{6}{10} + ... + \frac{23}{27}$

5. $1 + 3 + 5 + 7 + 9 + 11 + ... + 35$

Example 2. Write the sum in expanded form

1. $\sum_{i=4}^{9} \sqrt{i}$

2. $\sum_{i=0}^{4} 3^i$

3. $\sum_{i=1}^{6} \frac{1}{i+1}$

4. $\sum_{i=1}^{n} 2i$, here n is an integer

Theorem. If c is any constant (this means that c does not depend on i), then

1. $\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$
2. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
3. $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

- $\sum_{i=1}^{n} 1 = n$
- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

- $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n-1)}{r-1}$

Example 3. Evaluate

1. $\sum_{i=1}^{n} (3+2i)^2$

2. $\sum_{i=1}^{n} i(i+1)(i+2)$

3. $\sum_{i=1}^{n} (2i+2^i)$

Example 4. Find $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^2$