

Section 6.4 **The fundamental theorem of calculus.**

**Theorem.** Suppose  $f$  is continuous on  $[a, b]$ .

1. If  $g(x) = \int_a^x f(t)dt$ , then  $g'(x) = f(x)$ .
2.  $\int_a^b f(x)dx = F(b) - F(a) = F(x)|_a^b$ , where  $F$  is an antiderivative of  $f$ .

**Example 1.** Find the derivative of the function.

1.  $g(x) = \int_{\pi}^x \frac{1}{1+t^4} dt$

2.  $f(x) = \int_x^4 (2 + \sqrt{t})^8 dt$

3.  $y = \int_{\tan x}^{17} \sin(t^4) dt$

**Example 2.** Evaluate the integral.

1.  $\int_2^6 \frac{1 + \sqrt{y}}{y^2} dy$

2.  $\int_0^2 f(x)dx$ , where  $f(x) = \begin{cases} x^4 & 0 \leq x < 1 \\ x^5 & 1 \leq x \leq 2 \end{cases}$

**Example 3.** A particle moves along a line so that its velocity at time  $t$  is  $v(t) = t^2 - 2t - 8$ .

1. Find the displacement of the particle during the time period  $1 \leq t \leq 6$ .

2. Find the distance traveled during this time period.