Derivatives of vector functions

Definition. The **derivative of a vector function r**(t) **at a number** a, denoted by $\mathbf{r}\,'(a)$, is

$$\boxed{\mathbf{r}\,'(a) = \lim_{h\to 0}\frac{\mathbf{r}(a+h)-\mathbf{r}(a)}{h} = \lim_{t\to a}\frac{\mathbf{r}(t)-\mathbf{r}(a)}{t-a}}$$

if the limits exist.

If $\mathbf{r}(t) =< x(t), y(t) >$ is a vector function, then

$$\mathbf{r}\,'(t) = \lim_{h\to 0}\frac{\mathbf{r}(t+h)-\mathbf{r}(t)}{h} = \left\langle \lim_{h\to 0}\frac{x(t+h)-x(t)}{h}, \lim_{h\to 0}\frac{y(t+h)-y(t)}{h}\right\rangle =< x'(t), y'(t) >$$

if both $x'(t)$ and $y'(t)$ exist. Thus,

$$\boxed{\mathbf{r}'(t) =< x'(t), y'(t) >}$$

Example 1. Find the domain and the derivative of the vector function $\mathbf{r}(t) =< t^2-4, \sqrt{t-4} >$.

Example 2. Find a tangent vector of unit length for the curve given by $\mathbf{r}(t) = 2\sin t\mathbf{i} + 3\cos t\mathbf{j}$ at the point where $t = \pi/6$.

Definition. If $\mathbf{r}(t) =< x(t), y(t) >$ is a vector function representing the position of a particle at time t, then
velocity at time t is

$$\boxed{\mathbf{v}(t) = \mathbf{r}'(t) =< x'(t), y'(t) >}$$

speed at time t is

$$\boxed{s = |\mathbf{v}(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}}$$

Example 3. The vector function $\mathbf{r}(t) =< t, 25t - 5t^2 >$ represents the position of a particle at time t. Find the velocity and the speed at $t = 1$.