Section 1.5 **Inverse trigonometric functions.**

Definition. A function f with domain A is called **one-to-one function** if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

Horizontal line test. A function is one-to-one if and only if no horizontal line intersects its graph more that once.

Example 1. Determine which of the following functions is one-to-one:

a) $f(x) = x + 5$

b) $g(x) = x^2 - 2x + 5$

c) $h(x) = x^3 - 1$

d) $p(x) = x^4 + 5$

Definition. Let f be one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \Longleftrightarrow f(x) = y$$

for any y in B.

$$\text{domain of } f^{-1} = \text{range of } f$$
$$\text{range of } f^{-1} = \text{domain of } f$$

Let f be one-to-one function with domain A and range B. If $f(a) = b$, then $f^{-1}(b) = a$.

Cancellation equations

$$\boxed{\begin{array}{c} f^{-1}(f(x)) = x \text{ for every } x \in A \\ f(f^{-1}(x)) = x \text{ for every } x \in B \end{array}}$$

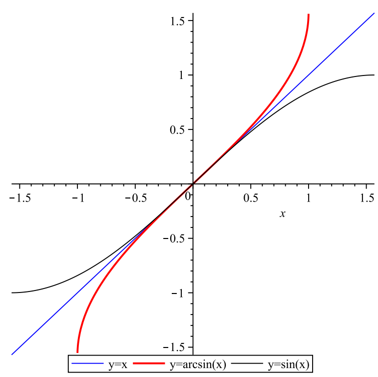
Inverse trigonometric functions

- **Inverse sine function**

$$\arcsin x = \sin^{-1} x = y \quad \Leftrightarrow \quad \sin y = x$$

DOMAIN $\quad -1 \leq x \leq 1$

RANGE $\quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



CANCELLATION EQUATIONS

$$\sin^{-1}(\sin x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\sin(\sin^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

Example 2. Find

1. $\sin^{-1}(0.5)$

2. $\sin^{-1}(\sin 1)$

3. $\sin(2\sin^{-1}\frac{3}{5})$

4. $\arcsin(\sin\frac{5\pi}{4})$

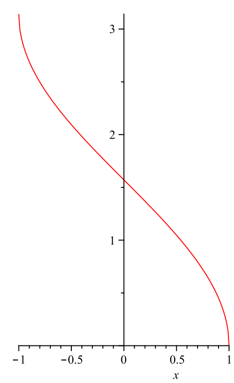
Example 3. Simplify $\tan(\sin^{-1}x)$

- **Inverse cosine function**

$$\arccos x = \cos^{-1} x = y \quad \Leftrightarrow \quad \cos y = x$$

DOMAIN $\quad -1 \leq x \leq 1$

RANGE $\quad 0 \leq y \leq \pi$



CANCELLATION EQUATIONS

$$\cos^{-1}(\cos x) = x \quad \text{for } 0 \leq x \leq \pi$$

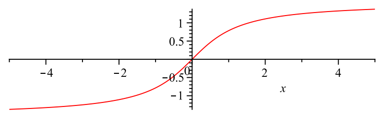
$$\cos(\cos^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

- **Inverse tangent function**

$$\arctan x = \tan^{-1} x = y \quad \Leftrightarrow \quad \tan y = x$$

DOMAIN $\quad -\infty \leq x \leq \infty$

RANGE $\quad -\frac{\pi}{2} < y < \frac{\pi}{2}$



CANCELLATION EQUATIONS

$$\tan^{-1}(\tan x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\tan(\tan^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty$$

$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

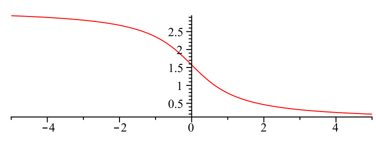
$$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

- **Inverse cotangent function**

$$\text{arccot} x = \cot^{-1} x = y \quad \Leftrightarrow \quad \cot y = x$$

DOMAIN $\quad -\infty \leq x \leq \infty$

RANGE $\quad 0 < y < \pi$



CANCELLATION EQUATIONS

$$\cot^{-1}(\cot x) = x \quad \text{for } 0 \leq x \leq \pi$$

$$\cot(\cot^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty$$

$$\lim_{x \to -\infty} \cot^{-1} x = 0$$

$$\lim_{x \to \infty} \cot^{-1} x = \pi$$

- **Other inverse trigonometric functions**

$$\csc^{-1} x = y \quad \Leftrightarrow \quad \csc y = x$$

DOMAIN $\quad |x| \geq 1$

RANGE $\quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$

$$\sec^{-1} x = y \quad \Leftrightarrow \quad \sec y = x$$

DOMAIN $\quad |x| \geq 1$

RANGE $\quad y \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$