A function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the **derivative of** f.

Example 1. Find the derivative of the function $f(x) = \dfrac{1-2x}{3+x}$ using the definition of the derivative.

Definition. A function f is **differentiable at** a if $f'(a)$ exists.

It is **differentiable on an open interval** (a, b) if it is differentiable at every number in the interval.

Example 2. Where is the function $f(x) = |x - 2|$ differentiable?

Theorem. If f is differentiable at a, then f is continuous at a

When is the function not differentiable at $x = a$?

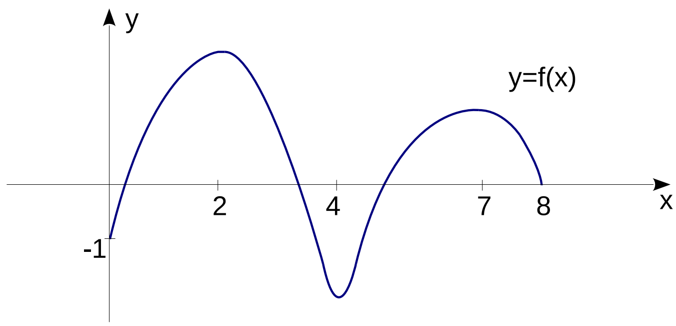
- f has a "corner" or "kink" at a
- f is discontinuous at a
- the curve $y = f(x)$ has a vertical tangent line at $x = a$

Besides being able to find the function $f(x)$ from the function $f(x)$, you can also determine the graph of $f(x)$ from the graph of $f(x)$.

Things to look for when sketching $f(x)$ from the given graph of $f(x)$:

- Values of slopes of tangent lines to $f(x)$
- Points where $f(x)$ has horizontal tangents
- Intervals where $f(x)$ is increasing or decreasing
- Places where $f(x)$ is leveling off

Example 3. Given the graph of $f(x)$ below, sketch the graph of $f'(x)$.



Higher derivatives.

Leibniz notations: $y'(x) = \frac{dy}{dx}$

Then

$$y''(x) = (y'(x))' = \frac{d^2y}{dx^2}$$
$$y'''(x) = (y''(x))' = \frac{d^3y}{dx^3}$$
$$y^{IV}(x) = y^{(4)}(x) = \frac{d^4y}{dx^4}$$
$$y^{(n)}(x) = \frac{d^ny}{dx^n}$$

Example 4. If $f(x) = 2x^2 - x^3$, find f', f'', f''', $f^{(4)}$.