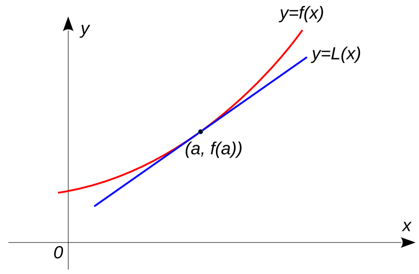
Section 3.10 **Linear approximations and differentials**

The equation of the tangent line to the graph of $f(x)$ at the point $(a, f(a))$ can be used to approximate values of f that are near a.



This is called the **linear approximation** or **tangent line approximation** of f at a

$$\boxed{f(x) \approx f(a) + f'(a)(x - a)}$$

The function

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$

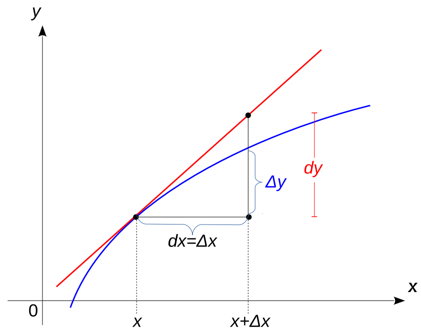
is called the **linearization** of f at a.

Example 1. Find the linearization $L(x)$ of the function $f(x) = \dfrac{1}{\sqrt{2+x}}$ at $a = 0$

Example 2. Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$.

Differentials.

We are using the notation $\dfrac{dy}{dx}$ to denote the derivative of y with respect to x, and it gives the slope of the tangent line at x. Now, we will actually view dy and dx as separate entities, called differentials.



Definition Let $y = f(x)$, where f is a differentiable function. Then the **differential** dx is an independent variable; that is dx can be given the value of any real number. The **differential** dy is then defined in terms of dx by the equation

$$\boxed{dy = f'(x)dx}$$

Example 3. Find dy if $y = x \tan x$.

Example 4.

(a.) Find dy if $y = \sqrt{1-x}$

(b.) Find the value of dy when $x = 0$ and $dx = .02$

Suppose that $f(a)$ is a known number and the approximate value is to be calculated for $f(a + \Delta x)$ where Δx is small. Then

$$\boxed{f(a + \Delta x) \approx f(a) + dy = f(a) + f'(a)\Delta x}$$

Example 3. Use differentials to find an approximate value for

(a.) $\sqrt{36.1}$

(b.) $\sin 59^\circ$