Section 3.1 **Derivatives of plynomials and exponentials functions.**

$$(C)' = 0,\ \ C \text{ is a constant}$$
$$(x)' = 1$$
$$(x^n)' = nx^{n-1} \text{ for any rational } n$$
$$(e^x)' = e^x$$

$(x^2)' = 2x$

$(x^3)' = 3x^2$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

Definition of the number e.
e is the number such that

$$\boxed{\lim_{h\to 0}\frac{e^h - 1}{h} = 1}$$

$$e \approx 2.7182818284590452$$

Differentiation formulas

Suppose c is a constant and both functions $f(x)$ and $g(x)$ are differentiable.

1. $(cf(x))' = cf'(x)$,
2. $(f(x) + g(x))' = f'(x) + g'(x)$,
3. $(f(x) - g(x))' = f'(x) - g'(x)$,

Example 1. Differentiate each function.

1. $f(x) = x^5 - 4x^3 + 2x - 3$

$f'(x) = (x^5)' - 4(x^3)' + 2(x)' - \cancel{(3)'}$ 0

$= 5x^4 - 4(3x^2) + 2$

$= \boxed{5x^4 - 12x^2 + 2}$

2. $f(x) = 3x^{2/3} - 2x^{5/2} + x^{-3}$

$f'(x) = 3 \cdot \frac{2}{3} x^{\frac{2}{3}-1} - 2 \cdot \frac{5}{2} x^{\frac{5}{2}-1} + (-3)x^{-3-1}$

$= \boxed{2x^{-1/3} - 5x^{3/2} - 3x^{-4}}$

3. $f(x) = x^2 \left(\sqrt[3]{x^2}\right) = x^2 \cdot x^{\frac{2}{3}} = x^{2+\frac{2}{3}} = x^{\frac{8}{3}}$

$f'(x) = \left(x^{\frac{8}{3}}\right)' = \frac{8}{3} x^{\frac{8}{3}-1} = \boxed{\frac{8}{3} x^{\frac{5}{3}}}$

4. $f(x) = \frac{2}{\sqrt[3]{x^2}} - \frac{1}{x\sqrt[3]{x}} = 2x^{-2/3} - \frac{1}{\underbrace{x \cdot x^{1/3}}_{x^{1+\frac{1}{3}} = x^{\frac{4}{3}}}} = 2x^{-2/3} - x^{-4/3}$

$$\begin{aligned} f'(x) &= 2\left(x^{-2/3}\right)' - \left(x^{-4/3}\right)' \\ &= 2\left(-\tfrac{2}{3}\right) x^{-2/3-1} - \left(-\tfrac{4}{3}\right) x^{-\frac{4}{3}-1} \\ &= \boxed{-\frac{4}{3} x^{-\frac{5}{3}} + \frac{4}{3} x^{-\frac{7}{3}}} \end{aligned}$$

5. $f(x) = x^{1.2} + e^{1.2}$

$$\begin{aligned} f'(x) = \left(x^{1.2}\right)' + \left(e^{1.2}\right)' &= 1.2\,x^{1.2-1} + 0 \\ &= \boxed{1.2\,x^{0.2}} \end{aligned}$$

6. $f(x) = x^e + e^x$

$$f'(x) = \underbrace{\left(x^e\right)'}_{\text{power function}} + \underbrace{\left(e^x\right)'}_{\text{exponential function}}$$

$$= \boxed{e\,x^{e-1} + e^x}$$

Example 2. Find an equation of the tangent line to the curve $y = x^4 + 1$ that is parallel to the line $32x - y = 15$.

$y = 32x - 15 \Rightarrow$ slope $= 32$

slope of tangent line $= (x^4+1)' = \underbrace{4x^3 = 32}$

$x^3 = 8 \Rightarrow x = 2.$

Tangent line @ $(a, f(a))$

$y - f(a) = f'(a)(x-a) \Rightarrow a = 2.$

$f(2) = 2^4 + 1 = 17$

$\boxed{y - 17 = 32(x-2)}$

Example 3. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases} \Rightarrow f'(x) = \begin{cases} 2x, & x \leq 2 \\ m, & x > 2 \end{cases}$$

Find the values for m and b that make f differentiable everywhere.

$f(x)$ must be continuous @ $x = 2$

$f'(x)$ must be continuous @ $x = 2$.

$f'(2^-) = f'(2^+)$

$f'(2)_{x<2} = 2(2) = 4$

$f'(2)_{x>2} = m$

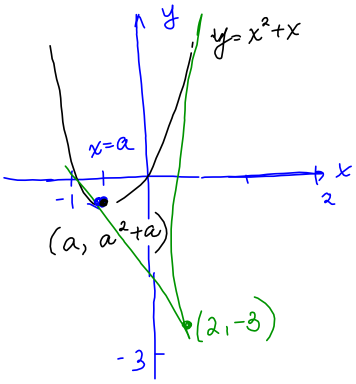
$\boxed{m = 4}$

update $f(x)$:

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ 4x + b, & x > 2 \end{cases}$$

$$\underbrace{\lim_{x \to 2^-} f(x)}_{2^2} = \underbrace{\lim_{x \to 2^+} f(x)}_{4(2)+b} \Rightarrow 4 = 8 + b \Rightarrow \boxed{b = -4}$$

Example 4. Find equations of both lines through the point (2,-3) that are tangent to the parabola $y = x^2 + x$.



Let the tangent passes through the point where $x = a$.

$$y(a) = a^2 + a$$

an equation of the tangent line through (a, a^2+a) is

$$y - \underbrace{y(a)}_{a^2+a} = \underbrace{y'(a)}_{2a+1}(x - a)$$

$$y'(x) = 2x + 1, \quad y'(a) = 2a + 1$$

$$y - (a^2 + a) = (2a+1)(x-a),$$ a is an unknown number

Plug in $x = 2$ and $y = -3$:

$$-3 - (a^2 + a) = (2a+1)(2-a)$$

$$-3 - a^2 - a = 4a - 2a^2 + 2 - a$$

$$2a^2 - 3a - 2 - 3 - a^2 - a = 0$$

$$a^2 - 4a - 5 = 0$$

$$(a-5)(a+1) = 0$$

$$a_1 = 5, \quad a_2 = -1$$

Tangent lines: $y - (25+5) = (10+1)(x-5)$

$$\boxed{y - 30 = 11(x-5)}$$

$$y - (1-1) = (2(-1)+1)(x+1)$$

$$\boxed{y = -(x+1)}$$