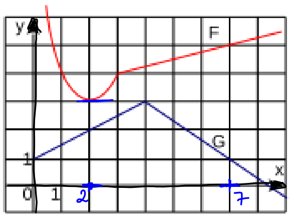
Section 3.2 **The product and quotient rules.**

$\boxed{(uvw)' = u'vw + uv'w + uvw'}$

Product Rule: $\boxed{(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)}$

Quotient Rule: $\boxed{\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}}$

Example 1. Let $P(x) = F(x)G(x)$ and $Q(x) = \frac{F(x)}{G(x)}$, where F and G are the functions whose graphs are given below.



$F(2) = 3$	$F'(2) = 0$
$G(2) = 2$	$G'(2) = \frac{1}{2}$

$F(7) = 5$	$F'(7) = \frac{1}{4}$
$G(7) = 1$	$G'(7) = -\frac{2}{3}$

Find

1. $P'(2)$

$P' = (FG)' = F'G + G'F$

$P'(2) = F'(2)G(2) + G'(2)F(2) = 0(2) + 3\left(\frac{1}{2}\right) = \frac{3}{2}$

2. $Q'(7)$

$Q'(x) = \left(\frac{F}{G}\right)' = \frac{F'G - G'F}{G^2}$

$Q'(7) = \frac{F'(7)G(7) - G'(7)F(7)}{G^2(7)} = \frac{\frac{1}{4}(1) - \left(-\frac{2}{3}\right)(5)}{1} = \boxed{\frac{1}{4} + \frac{10}{3}} = \frac{43}{12}$

Example 2. If $f(x) = e^x g(x)$, where $g(0) = 2$ and $g'(0) = 5$, find $f'(0)$.

$f'(x) = (e^x g(x))' = (e^x)' g(x) + g'(x) e^x$

$f'(x) = e^x g(x) + g'(x) e^x$

$f'(0) = \cancelto{1}{e^0}\,\cancelto{2}{g(0)} + \cancelto{5}{g'(0)}\,\cancelto{1}{e^0} = 2 + 5 = \boxed{7}$

Example 3. Differentiate.

1. $f(x) = (x+2\sqrt{x})e^x$

$$f'(x) = (x+2\sqrt{x})'e^x + (x+2\sqrt{x})(e^x)'$$
$$= \left(1+\cancel{2}\frac{1}{\cancel{2}}x^{-1/2}\right)e^x + (x+2\sqrt{x})e^x$$
$$= \left(1+\frac{1}{\sqrt{x}}\right)e^x + (x+2\sqrt{x})e^x$$

$$\lim_{h\to 0}\frac{e^h-1}{h} = 1 \Rightarrow e \approx 2.8273...$$

2. $f(x) = \dfrac{x^2-2}{2x+3}$

$$f'(x) = \frac{\overbrace{(x^2-2)'}^{2x}(2x+3)-(x^2-2)\overbrace{(2x+3)'}^{2}}{(2x+3)^2} = \frac{(2x)(2x+3)-(x^2-2)(2)}{(2x+3)^2}$$

3. $f(x) = \left(\dfrac{1}{x^2}+\dfrac{3}{x^4}\right)(x+5x^3)$ $= \dfrac{(x^2+3)\cancel{x}(1+5x^2)}{x^{\cancel{4}\,3}} = \dfrac{(x^2+3)(1+5x^2)}{x^3} = (x^2+3)(1+5x^2)x^{-3}$

$$f'(x) = \left[(x^2+3)(1+5x^2)x^{-3}\right]' = \overbrace{(x^2+3)'}^{2x}(1+5x^2)x^{-3} + (x^2+3)\underbrace{(1+5x^2)'}_{10x}x^{-3}$$
$$+ (x^2+3)(1+5x^2)\underbrace{(x^{-3})'}_{-3x^{-4}}$$
$$= \boxed{2x(1+5x^2)x^{-3} + (x^2+3)(10x)(x^{-3}) + (x^2+3)(1+5x^2)(-3x^{-4})}$$

Example 4. Find an equation of the tangent line to the curve $y = \dfrac{1+x}{1+e^x}$ at the point $\left(0, \underbrace{\tfrac{1}{2}}_{y(0)}\right)$.

$$y'(x) = \frac{(1+x)'(1+e^x) - (1+e^x)'(1+x)}{(1+e^x)^2} = \frac{1+\cancel{e^x} - e^x(\cancel{1}+x)}{(1+e^x)^2} = \frac{1-xe^x}{(1+e^x)^2}$$

@ $x=0$: $\quad y'(0) = \dfrac{1-0e^{0}}{(1+e^{0})^2} = \dfrac{1}{4}$

Tangent line: $\quad y - y(0) = y'(0)(x-0)$

$$\boxed{y - \frac{1}{2} = \frac{1}{4}x}$$